POWER SYSTEM STATE ESTIMATION : IMPLEMENTATION AND EVALUATION

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

By
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to the
DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

AUGUST, 1978

Dedicated to

AAJOBA

CERTIFICATE

Certified that the work entitled "Power System State Estimation: Implementation and Evaluation", by Mr. P.M. Jaltare has been carried out under my supervision and has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENT

I express my sincere thanks to Dr. R.P. Aggarwal whose valuable guidance and cooperation has encouraged me at every step. I feel very much obliged to Dr.M.A. Pai for his keen interest and continued help in this topic.

I also thank all my friends, 'C Block' friends in particular who helped me directly or indirectly in carrying out this work and made my stay here more enjoyable. I thank Vishwas Deshpande for the help he has been doing to me from the beginning of our stay here. Thanks are also due to Mr. K.N. Tewari for the fast typing he has done.

P. M. Jaltare

CONTENTS

CHAPTER 1

INTRODUCTION

State estimation in power systems is a data processing algorithm which estimates the true state of the system from the following sources of information:

1. Measurements of systems variables.
2. System model.
3. Apriori knowledge of certain variables known as pseudomeasurements.

The discrepencies in the estimated and true state of system are because of meter in-accuracies, communication errors errors in pseudomeasurements etc. Such an estimate is necessar for deciding various control policies to improve the performan of power system in both steady and dynamic states.

1.1 MOTIVATION

Many real time control schemes, in fact all of them, for static or quasistatic and dynamic performance of power systems are benifited by the state estimation. Some of them particularly dynamic performance schemes would be impossible without state estimation. Some of the real time control schen applied in steady or quasisteady state are:

1. Steady state security analysis and security monitoring.

2. Contingency evaluation.
3. Economic load dispatch.
4. Load frequency control.

The first two functions cannot be performed in absence of state estimation and the latter two functions work more reliably with state estimation for the quasisteady state being available. The information which is not directly contained in the state vector, could be calculated from the state vector upon interrogation.

The state estimation also enables displays which provides the dispatcher with the following information:

1. Estimates of lineflows, bus injection, voltage and bus angles etc. throughout the system.
2. Alarms to indicate system anamolies like information system malfunctioning etc. and their geographical location.
3. Security indices.
4. Fault location indication.

State estimate being more complete and accurate, its use for data logging is obvious.

Because of complexities, dynamic state estimation generally is confined to estimation of the state of individual generators under transient conditions. The need for estimation arises because of the fact that the variables of the synchronous machine state space models (for example, flux linkages, direct

and quadrature axis voltages, damper bar currents etc.), are not directly measurable. The state vector is built by measurable output variables of the synchronous machine by continuous or discrete time, linear or nonlinear observers. The state vector thus built is used to generate an optimum governer exciter control to improve the transient response of the generator.

1.2 SUMMARY OF VARIOUS CHAPTERS

Various methods for state estimation proposed in literature and some modifications in them have been discussed and implemented for a standard 14 bus IEEE system in Chapters 2, 3 and 4. All these methods work on the principle of weighted least squares estimation.

The basic lumped weighted least squares method and its fast decoupled version is given in Chapter 2. In addition, an estimator based on real power flow and voltage magnitude measurement only has been given. Such an estimator is particularly suited for Indian conditions. Chapter 3 discusses a very important state estimation algorithm which takes only line flow measurements into account. The properties of this algorithm which make it suitable for online applications are discussed. An approximate method for state estimation by solving independent equations is proposed in Chapter 4. In spite of being an approximate method, it has some plus points over other methods discussed in Chapters 2 and 3.

Detection and identification of inadvertant bad data coming in the algorithm due to communication failures etc. is the subject matter of Chapter 5. The capabilities of the detection and identification algorithm are demonstrated on the 14 bus IEEE system. The PV method of state estimation given in Chapter 2 was used for testing the algorithm.

A discrete time extended linear observer to estimate the transient state of a synchronous machine given in [23] is implemented in Chapter 6. Such observers are necessary in cases where a linear observer does not have adequate capability to estimate the transient state.

Chapter 7 briefly discusses the areas related to state estimation where a need for further work remains. Finally, a comparative study of the static state estimation algorithms given in Chapters 2,3 and 4 is made in Chapter 8 of evaluation and conclusion.

CHAPTER 2

LUMPED WLS AND FAST DECOUPLED STATE ESTIMATOR

The theory of statistical least squares estimation (linear estimation) is applied to power systems to produce best possible estimate of the true state from the information available which is contaminated by metering and communication errors. This estimate provides data base for many central control and dispatch functions as already discussed in Introduction. The equations encountered in power system state estimation are nonlinear. They are linearized and the linear estimation carried out iteratively till convergence is obtained. The computational time and storage requirements which are important considerations in any real time control scheme, are minimized by applying decoupling concept to state estimation.

It is also shown that state estimation can successfully be carried out with real power flow and voltage magnitude measurements only. These measurements are readily available hence it is convenient to carry out state estimation with them.

2.1 STATEMENT OF THE STATE ESTIMATION PROBLEM:

Complete information about the state of a power system is contained in the voltage magnitude and bus angles at all its buses with respect to slack bus. Hence the state vector

is defined as the voltage magnitude at all buses and bus angles with respect to slack bus. For a 'n' bus system the order of the state vector will thus be '2n-1'. The state vector in mathematical terms is defined as:

$$x^T = (V_1, V_2, \ldots, V_n; \theta_2, \theta_3, \ldots, \theta_n)$$

where V_i, $i = 1,..n$ are the voltage magnitudes and θ_i, $i = 2,..n$ are the bus angles with respect to slack bus.

In load flow studies also the solution vector is same as the state vector used in state estimation. However, numbers of equations is equal to the number of unknown quantities and the known quantities in forming the equations are assumed to be correctly known. However, in case of state estimation, many redundant measurements are made so as to be able to solve for the state vector even in the event of unavailability of a few important measurements and to facilitate filtering of measurement and communication errors present in the available information. The redundancy is defined as

$$\text{Redundancy} = \frac{\text{Number of measured quantities}}{\text{Order of the state vector.}}$$

The possible sets of measurements that can be made in a system of 'n' buses and 'l' lines is given in **Table 2.1.** It is seen from the table that the maximum figure of redundancy is $= (6\,l + 4\,n)/(2n - 1)$. It is desired to have a low redundancy from economic point of view. However, the

actual figure of redundancy depends upon technical considerations. It has been found for power systems that a figure between 1.5 to 2.5 is satisfactory dependent upon the system configuration and type of measurements made.

Table 2.1

Symbol	Physical quantity	Max. number of measurements
P_{ik}, Q_{ik}	Active and reactive power flows at both ends of the lines.	4 l
I_{ik}	Magnitude of current in both ends of the lines.	2 l
V_i	Magnitude of the bus voltage at all buses.	n
$Q_i P_i$	Reactive and active bus injections at all buses.	2 n
I_i	Magnitude of nodal current.	n

2.2 THE LUMPED WLS ESTIMATOR:

The measurements Z which could be a set or subset of quantities listed in Table 2.1 are nonlinear functions of the state vector, X, of the form:

$$Z = h(X) + \text{noise} \tag{2.1}$$

The noise is assumed to be zero-mean Gaussian random variable .

These equations are linearized at X^o and linear estimation theory applied directly to solve them. The linearization is done by Taylor series expansion of the function h(X) about X^o and truncation upto first order term as follows:

$$Z = h(X^o + \Delta X) \approx h(X^o) + h'(X^o)\Delta X \tag{2.2}$$

$$Z - h(X^o) = h'(X^o)\Delta X$$

$$\Delta Z = H(X^o)\Delta X \tag{2.3}$$

where $\Delta Z = Z - h(X^o)$ and $H(X^o) = h'(X^o)$.

As seen from equation (A.6) of Appendix, the solution to the linear equations (2.3) is given by

$$\Delta X = [\, H^T(X^o)\ W\ H(X^o)\ ]^{-1} [\, H^T(X^o)\ W] \Delta Z \tag{2.4}$$

The equation (2.4) can be written as

$$\Delta X = G^{-1} b \tag{2.5}$$

where $G = H^T(X^o) W\ H(X^o)$ and $b = H^T(X^o) W \Delta Z$.

G is called information matrix or gain matrix. It is symmetric. It has the properties of Gramm matrix when W is taken as identity.

The equation (2.5) is solved iteratively for ΔX, each time updating the value of X^o, till the $\| \Delta X \|_\infty$ is less than a specified limit.

2.3 MODEL FOR POWER SYSTEM NETWORK:

For simulation the following types of measurements were considered:

(1) Bus injections,

(2) Line flows,

(3) Bus voltage magnitudes.

The models h(X) for power system network which correlates the state vector and the above type of measurements are given below.

(a) Bus injections: The network is represented by bus admittance model. The bus injections are related to state vector by the following expressions,

$$P_p = |V_p| \sum_{q=1}^{n} (G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) |V_q| \qquad ($$

$$Q_p = | V_p | \sum_{q=1}^{n} (G_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) |V_q| \qquad (2.6)$$

$$p = 1,2,\ldots,n$$

where P_p - real power at bus p,

Q_p - reactive power at bus p,

V_p, V_q - voltage magnitudes at buses p and q,

θ_{pq} - bus angle θ_p - bus angle θ_q,

G_{pq} & V_{pq} - real and imaginary parts of the pq^{th} entry in Y_{Bus}.

(b) Line flows: A line or a off-nominal ratio transformer between two nodes, i and k is represented as a four terminal network by driving point and transfer admittance (Fig. 2.1).

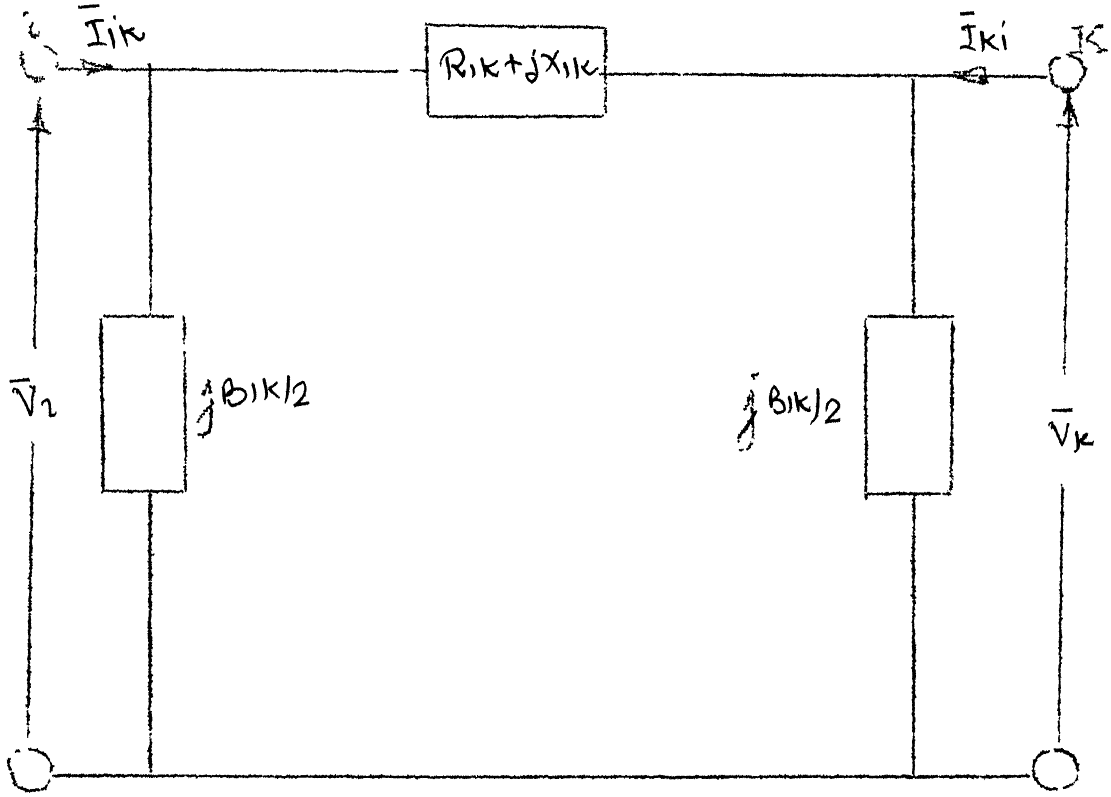

Fig 2.1.

A transmission line represented by two port network.

The figure is valid only for line. However it could be modified for a transformer. The node voltages $\bar{V}_i$ and $\bar{V}_k$ are related to currents by

$$\begin{bmatrix} \bar{I}_{ik} \\ \bar{I}_{ki} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{ii} & \bar{Y}_{ik} \\ \bar{Y}_{ki} & \bar{Y}_{kk} \end{bmatrix} \begin{bmatrix} \bar{V}_i \\ \bar{V}_k \end{bmatrix} \qquad (2.7)$$

The individual terms of the Y matrix are given by

$$\bar{Y}_{ii} = \frac{1}{R_{ik}+jX_{ik}} + j\,\frac{B_{ik}}{2}\ ; \quad \bar{Y}_{kk} = \frac{1}{R_{ik}+jX_{ik}} + j\,\frac{B_{ik}}{2}$$

$$\bar{Y}_{ik} = \bar{Y}_{ki} = -\,\frac{1}{R_{ik}+jX_{ik}} \qquad (2.8)$$

The relation between line flows and the voltages at the two ends of a line is

$$P_{ik}+jQ_{ik} = \bar{V}_i\bar{I}^*_{ik} = \bar{V}_i\ \bar{V}^*_i\ \bar{Y}^*_{ii} + \bar{V}_i\ \bar{V}^*_k\ \bar{Y}^*_{ik} \qquad (2.9)$$

The opposite end line flow

$$P_{ki} + jQ_{ki} = \bar{V}_k\bar{I}^*_{ki} = \bar{V}_k\ \bar{V}^*_i\ \bar{Y}^*_{ki} + \bar{V}_k\ \bar{V}^*_k\ \bar{Y}^*_{kk} \qquad (2.10)$$

If $\bar{Y}_{ii}$ is expressed as $(G_{ii}+jB_{ii})$ and $\bar{Y}_{ik}$ as $(G_{ik}+jB_{ik})$ the expression for line flows with voltages as polar quantities $V_i\angle\theta_i$ and $V_k\angle\theta_k$ becomes

$$P_{ik} = |V_i|^2\ G_{ii} + |V_i|\ |V_k|\ (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$$

$$Q_{ik} = -|V_i|^2\ B_{ii} + |V_i|\ |V_k|\ (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) \qquad (2.11)$$

where $\theta_{ik} = (\theta_i - \theta_k)$.

Similar expressions for P_{ki} and Q_{ki} can be written by interchanging i and k in the above expressions.

(c) Voltage magnitudes: This measurement directly gives the bus voltage magnitude. Hence the h(X) is a null matrix augmented by identity matrix,

$$Z = \begin{bmatrix} 0 & \vdots & I \end{bmatrix} \begin{bmatrix} \theta \\ \cdots \\ V \end{bmatrix}$$

2.4 ENTRIES OF THE JACOBIAN:

The measurement vector can be arranged as,

$$Z = \begin{bmatrix} Z_{active} \\ \\ Z_{reactive} \\ \\ Z_{voltage} \end{bmatrix} = \begin{bmatrix} \text{Active bus injections} \\ \text{Active line flows.} \\ \\ \text{Reactive bus in ections.} \\ \text{Reactive line flows.} \\ \\ \text{Voltage magnitudes} \end{bmatrix}$$

The Jacobian for each of these measurements is discussed below.

(a) Bus injection: The Jacobian for these measurement is nothing but the Jacobian used in Newton-Raphson's method to solve load flow problem in polar form with the only difference that there is no voltage controlled bus in this case. The state vector is taken as $[\Delta\theta, |\Delta V| / |V|]^T$ as it results into symmetry in certain terms of the Jacobian. To summarize the expressions:

$$J = \begin{bmatrix} H & N \\ M & L \end{bmatrix}$$

where

<u>for $p \neq q$</u>

$$H_{pq} = L_{pq} = |V_p|\,|V_q|\,(G_{pq}\sin\theta_{pq} - B_{pq}\cos\theta_{pq}) \tag{2.12}$$

$$N_{pq} = -M_{pq} = |V_p|\,|V_q|\,(G_{pq}\cos\theta_{pq} + B_{pq}\sin\theta_{pq}) \tag{2.13}$$

<u>for $p = q$</u>

$$H_{pp} = -Q_p - B_{pp}|V_p|^2 \tag{2.14}$$

$$L_{pp} = Q_p - B_{pp}|V_p|^2 \tag{2.15}$$

$$N_{pp} = P_p + G_{pp}|V_p|^2 \tag{2.16}$$

$$M_{pp} = P_p - G_{pp}|V_p|^2 \tag{2.17}$$

where P_p and Q_p are given by (2.6).

(b) Line flows: The entries of Jacobian corresponding to this measurement are found by taking partial derivatives of the real and reactive line flows of equation (2.11) with respect to bus angles and voltage magnitudes. We get eight terms viz.

$$\frac{\partial P_{ik}}{\partial \theta_i},\quad \frac{\partial P_{ik}}{\partial |V_i|},\quad \frac{\partial Q_{ik}}{\partial \theta_i},\quad \frac{\partial Q_{ik}}{\partial |V_i|},\quad \frac{\partial P_{ik}}{\partial \theta_k},\quad \frac{\partial P_{ik}}{\partial |V_k|},$$

$$\frac{\partial Q_{ik}}{\partial \theta_k},\quad \frac{\partial Q_{ik}}{\partial |V_k|}.$$

These derivatives are then to be multiplied by $|V|$ wherever necessary because the state vector is assumed to be $(\Delta\theta\ ,\ \Delta|V|/|V|)^T$. On performing the above operations, the following expressions are obtained:

$$\frac{\partial P_{ik}}{\partial \theta_i} = |V_i||V_k|(-G_{ik} \sin \theta_{ik} + B_{ik} \cos \theta_{ik}) \tag{2.18}$$

$$\left(\frac{\partial P_{ik}}{\partial |V_i|}\right)|V_i| = 2|V_i|^2 G_{ii}+|V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) \tag{2.19}$$

$$\frac{\partial Q_{ik}}{\partial \theta_i} = |V_i||V_k|(G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \tag{2.20}$$

$$\left(\frac{\partial Q_{ik}}{\partial |V_i|}\right)|V_i| = -2|V_i|^2 B_{ii}+|V_i||V_k|(G_{ik}\sin\theta_{ik}-B_{ik}\cos\theta_{ik}) \tag{2.21}$$

$$\frac{\partial P_{ik}}{\partial \theta_k} = |V_i||V_k|(G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) \tag{2.22}$$

$$\left(\frac{\partial P_{ik}}{\partial |V_k|}\right)|V_k| = |V_i||V_k|(G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \tag{2.23}$$

$$\frac{\partial Q_{ik}}{\partial \theta_k} = |V_i||V_k|(-G_{ik} \cos \theta_{ik} - B_{ik} \sin \theta_{ik}) \tag{2.24}$$

$$\left(\frac{\partial Q_{ik}}{\partial |V_k|}\right)|V_k| = |V_i||V_k|(G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}).$$

(c) Voltage magnitudes: The partial derivatives of the function relating voltage magnitude measurements and the voltage magnitudes, with respect to V will be unity and with respect to θ is taken as zero. Thus the Jacobian for this measurement becomes:

$$J = \left[\begin{array}{c} \text{Null matrix} \\ \hdashline \text{Diagonal matrix with} \\ \text{diagonal entries} \\ \text{equal to} \\ \text{voltage magnitudes} \end{array}\right] \tag{2.25}$$

To summarize the overall Jacobian for the three kinds of measurements discussed above:

$$H = \begin{array}{|c|c|}
\hline
\text{Load flow} & \text{Load flow} \\
H & N \\
\hline
\text{Line flow} & \text{Line flow} \\
\dfrac{\partial P_{ik}}{\partial \theta_i} \; ; \; \dfrac{\partial P_{ik}}{\partial |\theta_k|} & \left(\dfrac{\partial P_{ik}}{\partial |V_i|}\right)|V_i| \; ; \left(\dfrac{\partial P_{ik}}{\partial |V_k|}\right)|V_k| \\
\hline
\text{Load flow} & \text{Load flow} \\
M & L \\
\hline
\text{Line flow} & \text{Line flow} \\
\dfrac{\partial Q_{ik}}{\partial \theta_i} \; ; \; \dfrac{\partial Q_{ik}}{\partial \theta_k} & \left(\dfrac{\partial Q_{ik}}{\partial |V_i|}\right)|V_i| ; \; \left(\dfrac{\partial Q_{ik}}{\partial |V_k|}\right)|V_k| \\
\hline
\text{Zero} & \text{Diagonal matrix with entries } V_i \text{ in the diagonal} \\
\hline
\end{array} \qquad (2.26)$$

2.5 IMPLEMENTATION:

The lumped WLS method was implemented on the IEEE 14 bus test system. The results obtained from load flow analysis were used as input data. The weighting matrix was chosen to be identity. The flow-chart for implementation is given on next page. Results obtained are listed in Table 2.2.

FLOW CHART FOR LUMPED WLS METHOD:

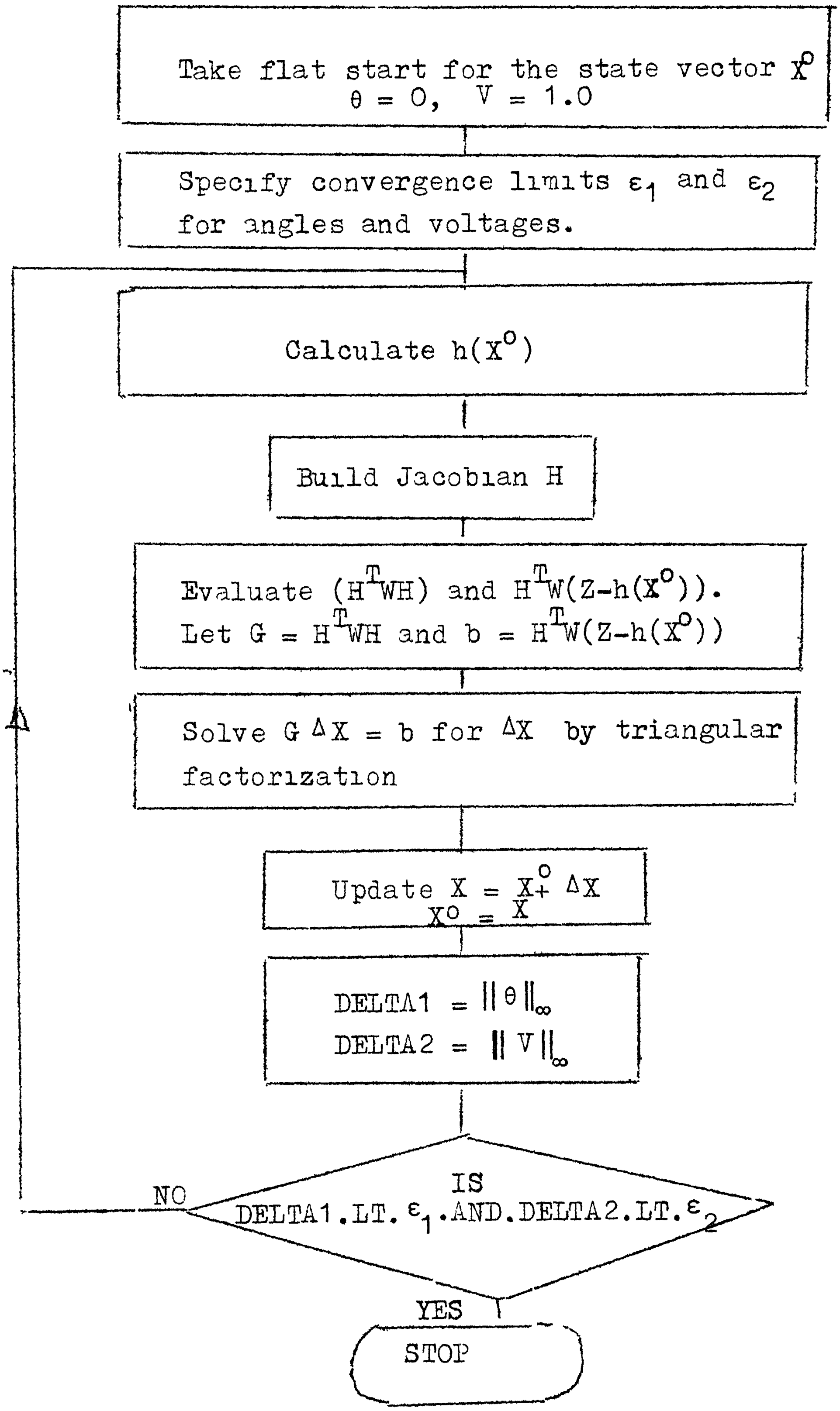

Table 2.2

RESULTS OF THE LUMPED WLS METHOD

BUS NO.	LOAD FLOW SOLUTION		WLS SOLUTION	
	Voltage	Angle	Voltage	Angle
1*	1.06	0.0	1.06	0.0
2	1.045	-4.98	1.03155	-4.65657
3	1.010	-12.72	0.99398	-12.34901
4	1.019	-10.33	1.00330	-9.96780
5	1.020	-8,78	1.00427	-8.37051
6	1.070	-14.22	1.05344	-13.91745
7	1.062	-13.37	1.05811	-13.14338
8	1.090	-13.36	1.08300	-13.12710
9	1.056	-14.94	1.05811	-14.75851
10	1.051	-15.10	1.05586	-14.99860
11	1.057	-14.79	1.05810	-14.75348
12	1.055	-15.07	1.05524	-15.18332
13	1.050	-15.16	1.04792	-15.16305
14	1.036	-16.04	1.04362	-16.09178

Redundancy = 3.0

No. of iterations = 4

Total time taken = 133 sec.

ε_1 = 0.0005

ε_2 = 0.0001=

* indicates slack bus.

2.6 THE FAST DECOUPLED ESTIMATOR (FDS):

The concept of P-θ, Q-V decoupling used in load flow studies is extended to state estimation. The basic work in this area is done in ref.[2 7]. The information matrix G of eq.(2.5) is approximated by two submatrices each corresponding to the respective decoupled loops. This reduces the storage and also the total time taken to converge, though the total number of iterations are more than that required by the lumped WLS estimator. The reduction of computational time is due to the fact that the submatrices derived from the gain matrix G are treated as constants. They need not be triangularized every iteration.

The measurement vector Z arranged in active and reactive groups is expressed in terms of state vector as

$$Z = \begin{bmatrix} Z_{active} \\ \hline Z_{reactive} \end{bmatrix} = \begin{bmatrix} f(V,\ \theta) \\ \hline g(V,\ \theta) \end{bmatrix} = h(\theta\ ,\ V) \qquad (2.27)$$

The Jacobian H(X) of the function h(X) is given by

$$\frac{dh}{dX} = \left[\frac{\partial h}{\partial \theta} \quad \frac{\partial h}{\partial V} \right] = \left[\begin{array}{c|c} \partial f/\partial\theta & \partial f/\partial V \\ \hline \partial g/\partial\theta & \partial g/\partial V \end{array} \right] = \left[\begin{array}{c|c} H_{11} & H_{12} \\ \hline H_{21} & H_{22} \end{array} \right] \qquad (2.28)$$

The information matrix G of eq.(2.5) is

$$G(\theta,V) = \left[\begin{array}{c|c} \frac{\partial f}{\partial\theta}^T R_a^{-1} \frac{\partial f}{\partial\theta} + \frac{\partial g}{\partial\theta}^T R_r^{-1} \frac{\partial g}{\partial\theta} & \frac{\partial f}{\partial\theta}^T R_a^{-1} \frac{\partial f}{\partial V} + \frac{\partial g}{\partial\theta}^T R_r^{-1} \frac{\partial g}{\partial V} \\ \hline \frac{\partial f}{\partial V}^T R_a^{-1} \frac{\partial f}{\partial\theta} + \frac{\partial g}{\partial V}^T R_r^{-1} \frac{\partial g}{\partial\theta} & \frac{\partial f}{\partial V}^T R_a^{-1} \frac{\partial f}{\partial V} + \frac{\partial g}{\partial V}^T R_r^{-1} \frac{\partial g}{\partial V} \end{array} \right] \qquad (2.29)$$

where R_a and R_r are diagonal matrices and

$$W^{-1} = R = \begin{array}{|c|c|} \hline R_a & 0 \\ \hline 0 & R_r \\ \hline \end{array}$$ W is the weighting matrix.

The above information matrix $G(\theta, V)$ is approximated by

$$G(\theta, V) = \begin{array}{|c|c|} \hline \frac{\partial f^T}{\partial \theta} R_a^{-1} \frac{\partial f}{\partial \theta} & 0 \\ \hline 0 & \frac{\partial g^T}{\partial V} R_r^{-1} \frac{\partial g}{\partial V} \\ \hline \end{array} \qquad (2.30a)$$

by assuming that the matrices H_{11} and H_{22} are large compared to H_{12} and H_{21}, **in eqn.(2.28) for information matrix only.**

A further simplification in the information matrix computation is made by calculating it once at the previously estimated value of X or flat starting value of X, i.e. $V=(1,1\ldots1)^T$ and $\theta = (0,0,\ldots0)^T$. The information matrix then becomes constant independent of θ and V.

$$G = \begin{array}{|c|c|} \hline A & 0 \\ \hline 0 & B \\ \hline \end{array} \qquad (2.30b)$$

Thus the iteration scheme becomes

$$\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} (\Delta X) = H^T R^{-1}(Z-h(\theta, V)) \qquad (2.31)$$

or $$A(\Delta\theta) = [H_{11}^T(\theta, V), H_{21}^T(\theta, V)] R^{-1}(Z-h(\theta, V)) \qquad (2.32)$$

$$B(\Delta V) = [H_{12}^T(\theta, V), H_{22}^T(\theta, V)] R^{-1}(Z-h(\theta, V)) \qquad (2.33)$$

The equations (2.32) and (2.33) can be solved iteratively to get the optimal solution of state vector X. The two matrices A

and B are symmetric and sparse in nature which need be factorized only once.

2.7 IMPLEMENTATION:

The FDS algorithm was implemented on the IEEE 14 bus test system. Results are listed in Table 2.3 along with the true solution obtained by load flow. The flow chart for implementation is given is given below.

FLOW CHART FOR FDS ALGORITHM IMPLEMENTATION:

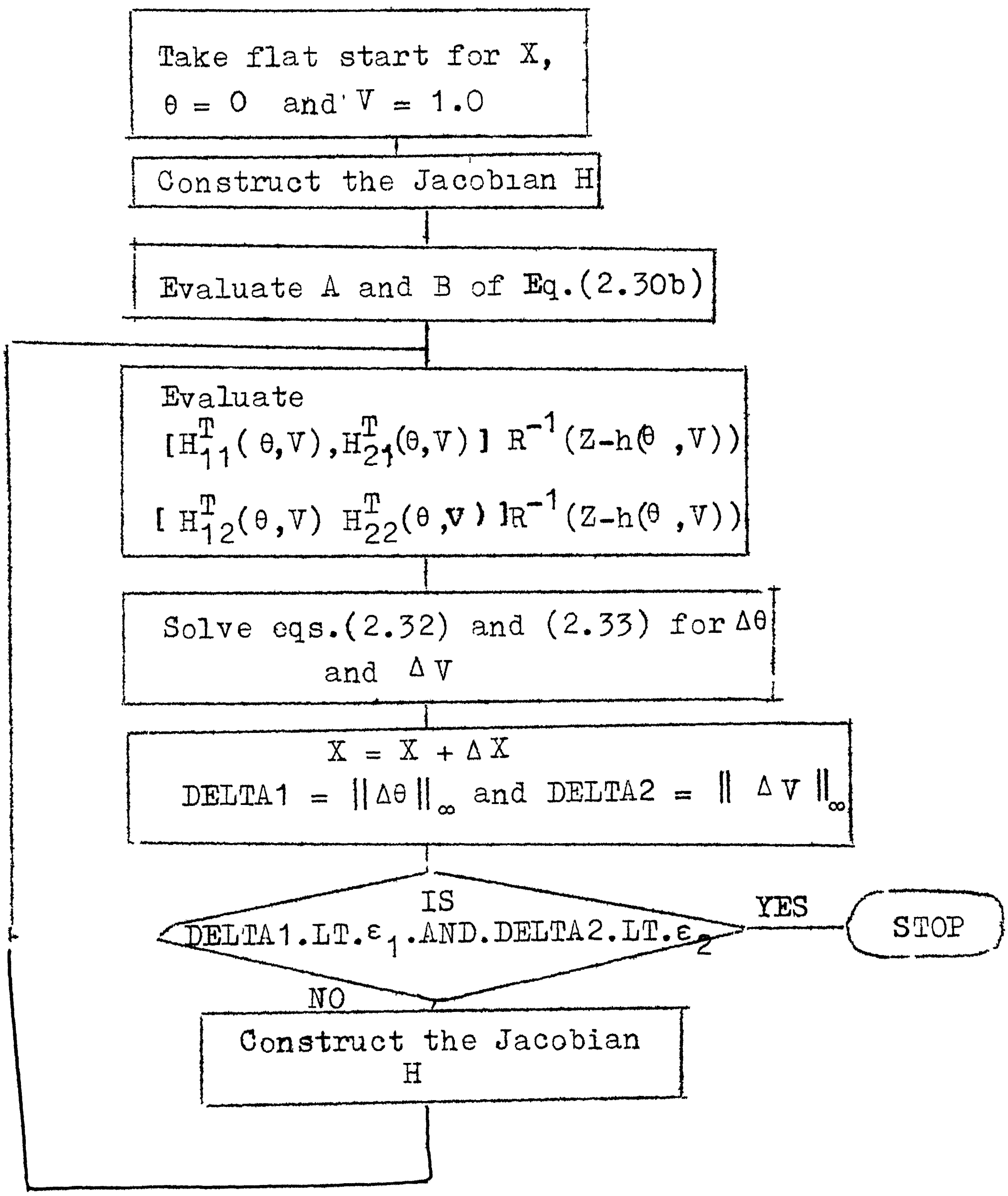

Table 2.3

RESULTS OF FDS METHOD

Bus No.	Load flow solution		FDS solution	
	Voltage	Angle	Voltage	Angle
1*	1.06	0.0	1.06	0.0
2	1.045	-4.98	1.0316	-4.6561
3	1.010	-12.72	0.9942	-12.3448
4	1.019	-10.33	1.0036	-9.9639
5	1.020	-8.78	1.00456	-8.3675
6	1.070	-14.22	1.0538	-13.9032
7	1.062	-13.37	1.0586	-13.1329
8	1.090	-13.36	1.0835	-13.1161
9	1.056	-14.94	1.0586	-14.7456
10	1.051	-15.10	1.0564	-14.9851
11	1.057	-14.79	1.05862	-14.7393
12	1.055	-15.07	1.0556	-15.1654
13	1.050	-15.16	1.08484	-15.1468
14	1.036	-16.04	1.04419	-16.07698

Redundancy = 3.0

No. of iterations = 12

Total time taken = 99.66 sec.

ε_1 = 0.002

ε_2 = 0.001

* indicates slack bus.

2.8 ACCELERATION OF CONVERGENCE:

The above approximations are advantageous in terms of storage and also the total time required for convergence. The only short coming is that it takes relatively more number of iterations to converge. Attempts have been made to minimize the number of iterations for such a fast decoupled method [27]. The scheme suggested in [27] is as follows:

The iterative scheme is written as

$$\theta_{i+1} = \theta_i + S_i \tag{2.34}$$

$$V_{i+1} = V_i + \tau_i \tag{2.35}$$

where
$$AS_i = -\tfrac{1}{2} \frac{\partial J^T}{\partial \theta} (\theta_i, V_i) \tag{2.36}$$

$$B \tau_i = -\tfrac{1}{2} \frac{\partial J^T}{\partial V} (\theta_i, V_i) \tag{2.37}$$

The convergence is accelerated by choosing an optimum step size α_i and β_i as follows:

$$\theta_{i+1} = \theta_i + \alpha_i S_i \tag{2.38}$$

$$V_{i+1} = V_i + \beta_i \tau_i \tag{2.39}$$

The choice of α_i and β_i should be theoretically made such that

$$J(\theta_i + \alpha_i S_i, V_i) = \min_{\alpha \geq 0} \{ J(\theta + \alpha S_i , V_i) \} \tag{2.40}$$

and

$$J(V_i + \beta_i \tau_i, V_i) = \min_{\beta \geq 0} \{ J(\theta + \beta \tau_i , V_i) \} \tag{2.41}$$

α_i and β_i can be found out by a linear search or curve fitting. A simple scheme to choose α_i and β_i can be setting

$$\alpha_{i+1} = \alpha_i \quad \text{if } J(\theta_{i+1}, V_i) < J(\theta_i, V_i) \tag{2.42}$$

and

$$\alpha_{i+1} = \alpha_i/2 \text{ if } J(\theta_{i+1}, V_i) \geq J(\theta_i, V_i) \tag{2.43}$$

Similar scheme can be used for β_i. This simple scheme is reported to have reduced the number of iteration by 20% in[27] . ~~Recently once such scheme has been suggested in~~

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2.9 STATE ESTIMATION BASED ON REAL POWER FLOW IN LINES AND VOLTAGE MAGNITUDES ONLY (PV) :

State estimation can successfully be carried out with real power flows in lines and voltage magnitude measurements only. These kinds of measurements are readily available and state estimator based on these measurements will particularly be suited for Indian conditions.

The derivation of algorithm directly follows from eqn.(A.6) of Appendix . The Jacobians for these kinds of measurements are given in Section 2.4b and 2.4c. The overall Jacobian will have the following form:

H =

Line flow $\frac{\partial P_{ik}}{\partial \theta_i}$, $\frac{\partial P_{ik}}{\partial \theta_k}$	Line flow $(\frac{\partial P_{ik}}{\partial \lvert V_i \rvert}) \lvert V_i \rvert$, $(\frac{\partial P_{ik}}{\partial \lvert V_k \rvert}) \lvert V_k \rvert$
Null	Diagonal matrix with entries $\lvert V_i \rvert$ on the diagonal

Iterative scheme given in eqn.(2.4) is then used to solve the state vector.

2.10 IMPLEMENTATION:

The PV state estimation was carried out for the IEEE 14 bus system. Results are tabulated in Table 2.4.

Table 2.4

RESULTS OF PV METHOD

Bus No.	Load flow solution		PVO solution	
	Voltage	Angle	Voltage	Angle
1*	1.06	0.0	1.06	0.0
2	1.045	-4.98	1.0437	-4.965
3	1.010	-12.72	1.0105	-12.727
4	1.019	-10.33	1.0217	-10.398
5	1.020	-8.78	1.0180	-8.762
6	1.070	-14.22	1.0782	-14.932
7	1.062	-13.37	1.0582	-13.621
8	1.090	-13.36	1.090	-13.621
9	1.056	-14.94	1.0506	-15.227
10	1.051	-15.10	1.0503	-15.488
11	1.057	-14.79	1.0565	-15.229
12	1.055	-15.07	1.055	-15.563
13	1.050	-15.16	1.0498	-15.610
14	1.036	-16.04	1.0359	-16.495

Redundancy = 1.27

No. of iterations = 4

Total time taken = 86.68 sec.

$\varepsilon_1 = 0.0005$

$\varepsilon_2 = 0.0001$

* indicates slack bus.

Table 2.4

RESULTS OF PV METHOD

Bus No.	Load flow solution		PVO solution	
	Voltage	Angle	Voltage	Angle
1*	1.06	0.0	1.06	0.0
2	1.045	-4.98	1.0437	-4.965
3	1.010	-12.72	1.0105	-12.727
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6	1.070	-14.22	1.0782	-14.932
7	1.062	-13.37	1.0582	-13.621
8	1.090	-13.36	1.090	-13.621
9	1.056	-14.94	1.0506	-15.227
10	1.051	-15.10	1.0503	-15.488
11	1.057	-14.79	1.0565	-15.229
12	1.055	-15.07	1.055	-15.563
13	1.050	-15.16	1.0498	-15.610
14	1.036	-16.04	1.0359	-16.495

Redundancy = 1.27

No. of iterations = 4

Total time taken = 86.68 sec.

ε_1 = 0.0005

ε_2 = 0.0001

* indicates slack bus.

CHAPTER 3

THE LINE ONLY ALGORITHM ('LO' ALGORITHM)

A very efficient state estimation algorithm has been developed in [34] which takes into account only line flow measurements. It is superior to lumped WLS and FDS algorithm in the following respects:

(1) Faster convergence.

(2) Simplicity of the algorithm and ease with which changes in network configuration or metering location can be taken into account.

(3) Lesser storage requirements.

However this algorithm has lesser flexibility for taking into account differing meter accuracies compared to lumped WLS and FDS algorithms. A further sacrifice in the flexibility to incorporate differing meter accuracies pays in terms of reduced storage when line flows in both ends of all measured lines are available.

3.1 THE AEP LINE ONLY ALGORITHM

A line between buses i and k is represented by a four terminal equivalent model as given in Section 2.3b of Chapter 2. Equations (2.7), (2.9) and (2.10) are repeated here for easy reference.

$$\begin{bmatrix} \bar{I}_{ik} \\ \bar{I}_{ki} \end{bmatrix} = \begin{bmatrix} \bar{Y}_{ii} & \bar{Y}_{ik} \\ \bar{Y}_{ki} & \bar{Y}_{kk} \end{bmatrix} \begin{bmatrix} \bar{V}_i \\ \bar{V}_k \end{bmatrix} \tag{2.7}$$

$$P_{ik}+jQ_{ik} = \bar{V}_i\ \bar{I}_{ik}^* = \bar{V}_i\ \bar{V}_i^*\ \bar{Y}_{ii}^* + \bar{V}_i\ \bar{V}_k^*\ \bar{Y}_{ik}^* \tag{2.9}$$

$$P_{ki}+jQ_{ki} = \bar{V}_k\ \bar{I}_{ki}^* = \bar{V}_k\ \bar{V}_i^*\ \bar{Y}_{ki}^* + \bar{V}_k\ \bar{V}_k^*\ \bar{Y}_{kk}^* \tag{2.10}$$

The terms have already been defined in Sec.2.3 of Chapter 2. The measured quantities are real and reactive line flows from bus i to k or k to i or both. These measured quantities are translated into voltage difference between the two nodes i and k. Line flow $P_{ik}+jQ_{ik}$ translated into $\bar{V}_i-\bar{V}_k$ and line flow $P_{ki}+jQ_{ki}$ into $\bar{V}_k-\bar{V}_i$ by the following relations:

$$\bar{V}_i-\bar{V}_k = \bar{V}_i\ \frac{\bar{Y}_{ik}+\bar{Y}_{ii}}{\bar{Y}_{ik}} - \frac{P_{ik}-jQ_{ik}}{\bar{V}_i^*\ \bar{Y}_{ik}} \tag{3.1}$$

and

$$\bar{V}_k-\bar{V}_i = \bar{V}_k\ \frac{\bar{Y}_{ki}+\bar{Y}_{kk}}{\bar{Y}_{ki}} - \frac{P_{ki}-jQ_{ki}}{\bar{V}_k^*\ \bar{Y}_{ki}^*} \tag{3.2}$$

These translated quantities are treated as measured quantities in further derivation. Call the complex vector of the translated measurements as $\bar{U}_h$. The function $h(\bar{X})$ which correlates $\bar{U}_h$ and $\bar{X}$ is

$$\bar{U}_h = h(\bar{X}) = B_1\bar{X} - B_2\bar{V}_R \tag{3.3}$$

where B_1 is the bus incidence matrix and B_2 is the column of element node incidence matrix which is deleted to obtain bus

incidence matrix B_1 and $\bar{V}_R$ is the complex slack bus voltage assumed to be correctly known.

To apply equation (A.6) of Appendix to estimate the state vector from the above translated measurements the Jacobian $H(\bar{X})$ is calculated by differentiating equation (3.3) with respect to $\bar{X}$;

$$H = B_1 \tag{3.4}$$

The Jacobian is constant. Thus the state estimate is obtained by iteratively applying equation (A.6) of Appendix as follows

$$\Delta\bar{X} = (B_1^T W B_1)^{-1} (B_1^T W) (\bar{U}_h - h(\bar{X}))$$

$$= (B_1^T W B_1)^{-1} (B_1^T W) (\bar{U}_h - B_1\bar{X} + B_2 \bar{V}_R) \tag{3.5}$$

where W is the weighting matrix for measurements. It should be noted that equal weights are assigned to P & Q measurements corresponding to individual line.

Equation (3.5) is simplified by expanding the expression as

$$\Delta\bar{X} = (B_1^T W B_1)^{-1} (B_1^T W) (\bar{U}_h + B_2 \bar{V}_R) - \bar{X}$$

or

$$\bar{X}_{new} = (B_1^T W B_1)^{-1} (B_1^T W)(\bar{U}_h + B_2 \bar{V}_R) \tag{3.6}$$

as $\bar{X}_{new}$ is $\bar{X} + \Delta\bar{X}$.

The equation (3.6) is solved iteratively **replacing** $\bar{X}$ by X_{new} until convergence is obtained.

3.2 MODIFICATION OF AEP ALGORITHM TO TAKE INTO ACCOUNT EFFICIENTLY BOTH END LINE FLOWS

When information about both end line flows in all the measured lines are available the algorithm of Section 3.1 can be modified which will reduce the size of B_1 matrix to half of what will normally be required by equation (3.6). The assumption made here is that weighting factor for the four measurements, viz, P and Q at both ends of a line are equal. This approximation is worth considering as it results in significant reduction in storage.

The derivation is as follows:

Let $\bar{U}_z$ be the vector of complex element voltages translated from measurement $P_{ki}+jQ_{ki}$ using equation (2.10) and $\bar{U}_h$ as described earlier be calculated from measurements $P_{ik}+jQ_{ik}$ using equation (2.9). We define two functions $h_1(\bar{X})$ and $h_2(\bar{X})$ similar to equation (3.3) relating state vector $\bar{X}$ with $\bar{U}_h$ and $\bar{U}_z$ respectively:

$$h_1(\bar{X}) = B_1\,\bar{X} - B_2\,\bar{V}_R \tag{3.7}$$

and $$h_2(\bar{X}) = -B_1\,\bar{X} - B_2\,\bar{V}_R \tag{3.8}$$

The Jaconian for these functions are

$$H_1 = B_1 \tag{3.9}$$

$$H_2 = -B_1 \tag{3.10}$$

Equation (A.6) of Appendix is modified to take into account n different types of measurements as

$$\bar{X} = \left(\sum_{i=1}^{n} H_i^T W_i H_i \right)^{-1} \left[\sum_{i=1}^{n} H_i^T W_i (Z_i - h_i(\bar{X})) \right]$$

where H_i - Jacobian corresponding to ith type of measurement,

W_i - Weighting matrix corresponding to ith type of measurement Z_i,

$h_i(\bar{X})$- Model for ith type of measurement Z_i.

Considering $\bar{U}_h$ and $\bar{U}_z$ to be two different types of measurements with equal weighting matrices we get,

$$\bar{X} = (B_1^T W B_1 + B_1^T W B_1)^{-1} [\, B_1^T W(\bar{U}_h - B_1\bar{X} + B_2\bar{V}_R) - B_1^T W(\bar{U}_z + B_1\bar{X} + B_2\bar{V}_R) \,]$$

$$= (2B_1{}^T W B_1)^{-1} [\, B_1^T W(\bar{U}_h - \bar{U}_z) - 2B_1^T W B_1 \bar{X} + B_1^T W(2B_2\bar{V}_R) \,]$$

$$= (B_1^T W B_1)^{-1} [\, B_1^T W(\frac{\bar{U}_h - \bar{U}_z}{2}) + B_1^T W B_2 \bar{V}_R \,] - \bar{X}$$

$$\bar{X}_{new} = (B_1^T W B_1)^{-1} [B_1^T W(\frac{\bar{U}_h - \bar{U}_z}{2}) + B_1^T W B_2 \bar{V}_R \,] \qquad (3.11)$$

Equation (3.11) is solved iteratively till convergence is obtained.

In order to have a complete state vector, it is necessary to have meters placed at least on lines that form tree of the whole network. When algorithm of equation (3.11) is being used, we have to have both end line flows measured. In case one end line flow is missing, the missing measurement is put equal to the other end line flow available. Thus the minimum figure of redundancy for the algorithm of equation(3.11) is 4n/2(n-1) (approximately 2.0) for an n bus l line system.

It is possible to apply this method to parts of the network if each such part has one reference bus.

3.3 IMPLEMENTATION

The 'LO' algorithm discussed above are very simple to be programmed. The gain matrix $(B_1^T W B_1)$ can be built just by inspecting the metring configuration as Y_{Bus} is built. It is very sparse. Its structure is same as that of Y_{Bus} of the metered part of the network. Off-diagonal entries are either 1.0 or -1.0 when W is taken as identity. The following algorithm builds the gain matrix, when W is identity. Following is the description of variables used.

(a) For an n+1 bus and l line system starting end index of lines are stored in vector NX(I) and terminating end index in NY(I) for I = 1 to l. (n+1)th bus is slack bus.

(b) The product $(B_1^T B_1)$ is stored in B.

Algorithm for finding $(B_1^T B_1)$:

Step 1: Initialize B(i,j) = 0, i = 1,...n, j = 1,...n, set KX = 0, i = 1.

Step 2: Set KX = KX+1 , KY=0.

Step 3: If NX(i) = KX go to step 4 else if NY(i)= KX go to step 6 else go to step 8.

Step 4: KY = KY+1, n1 = NY(i).

Step 5: If n1 ≠ n+1 then B(KX,n1) = -1.0 ; go to step 8.

Step 6: KY = KY+1, n1 = NX(i).

Step 7: If n1 ≠ n+1 then B(KX,n1) = -1.0 ; go to step 8.

Step 8: $i = i+1$.

Step 9: If i.LE.n go to step 3.

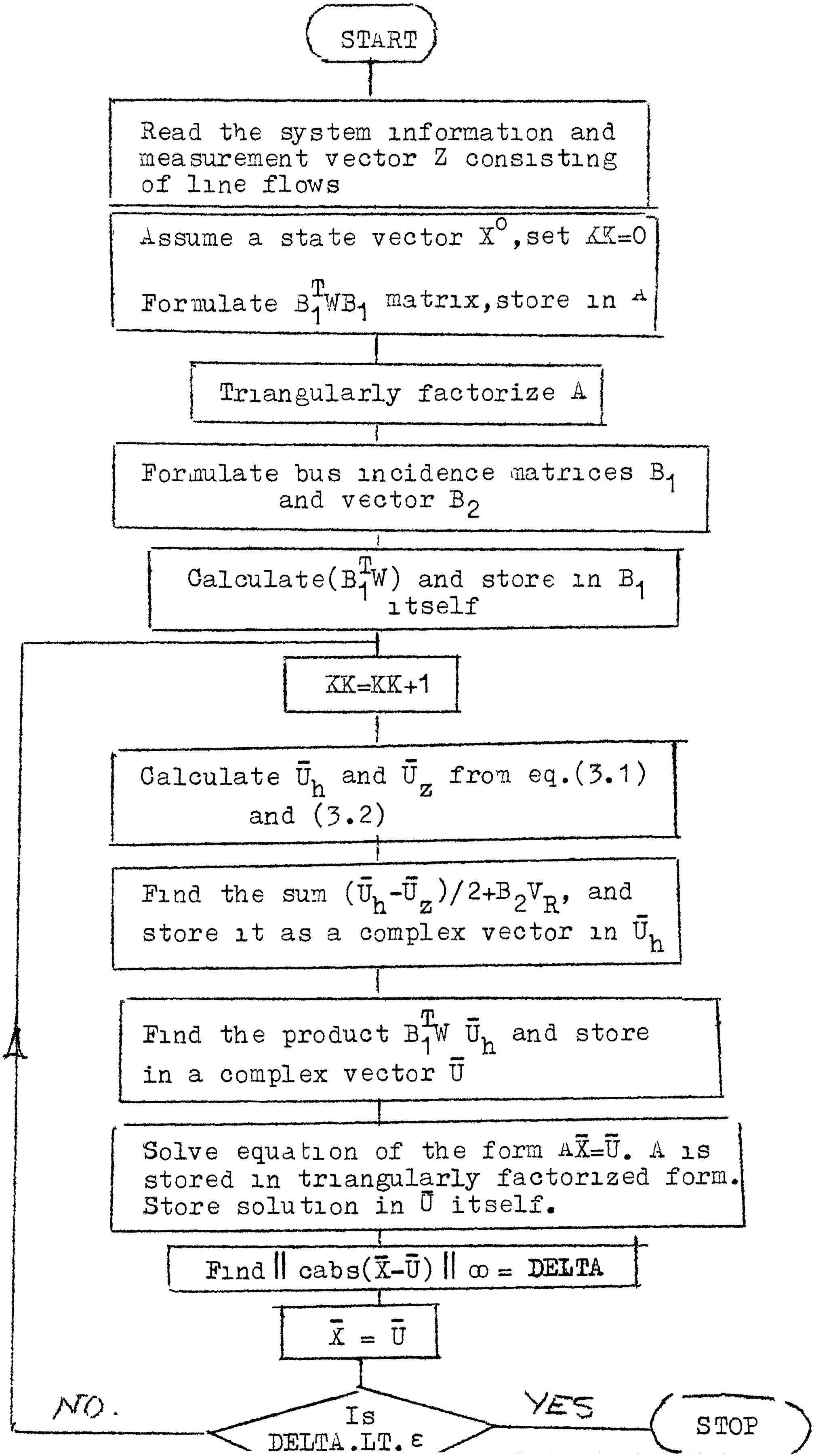
Step 10: B(KX, KX) = float(KY).

Step 11: If KX.LE.n go to step 2.

Step 12: Stop.

The matrix $B_1^T W$ also need not be stored explicitly as sufficient information is contained in arrays NX and NY to calculate the product of $B_1^T W$ with an array.

Flow chart for implementation of the 'LO' method is given on next page.


START
Read the system information and measurement vector Z consisting of line flows
Assume a state vector X^0, set KK=0
Formulate $B_1^T W B_1$ matrix, store in A
Triangularly factorize A
Formulate bus incidence matrices B_1 and vector B_2
Calculate($B_1^T W$) and store in B_1 itself
KK=KK+1
Calculate $\bar{U}_h$ and $\bar{U}_z$ from eq.(3.1) and (3.2)
Find the sum $(\bar{U}_h-\bar{U}_z)/2+B_2V_R$, and store it as a complex vector in $\bar{U}_h$
Find the product $B_1^T W \bar{U}_h$ and store in a complex vector $\bar{U}$
Solve equation of the form $A\bar{X}=\bar{U}$. A is stored in triangularly factorized form. Store solution in $\bar{U}$ itself.
Find $\|$ cabs($\bar{X}-\bar{U}$) $\|_\infty$ = DELTA
$\bar{X} = \bar{U}$
Is DELTA.LT. ε
NO.
YES
STOP

3.4 COMPUTER RESULTS

The algorithm (3.3) was implemented on the IEEE 14 bus test system. The results obtained and details of the algorithm are listed in Table 3.1.

Table 3.1

Bus No.	Load flow solution		'LO' algorithm	
	Voltage	Angle	Voltage	Angle
1*	1.06	0.0	1.06	0.0
2	1.045	-4.98	1.043	-4.98
3	1.010	-12.72	1.007	-12.757
4	1.019	-10.33	1.015	-10.356
5	1.020	- 8.78	1.022	- 8.770
6	1.070	-14.22	1.002	-14.922
7	1.062	-13.37	1.029	-13.628
8	1.090	-13.36	1.058	-13.628
9	1.056	-14.94	1.014	-15.402
10	1.051	-15.10	1.009	-15.570
11	1.057	-14.79	1.015	-15.246
12	1.055	-15.07	0.993	-15.803
13	1.05	-15.16	0.995	-15.809
14	1.036	-16.04	0.986	-16.689

Redundancy = 2.61

Total No.of iteration = 5

Time taken = 19.7 sec.

ε = 0.00009

* indicates slack bus.

CHAPTER 4

STATE ESTIMATION BY INDEPENDENT EQUATION SOLUTION

A method is proposed for state estimation which makes bus-by-bus sequential estimate of the state vector thereby obviating large matrix storages. Also this method is faster than the methods discussed in Chapters 2 and 3. The bus-by-bus sequential estimate is obtained by splitting the lumped equations into independent equations. Accuracy of such an independent equation solution will depend upon network topology and instrument location. Such a technique when applied to a radial system may give unacceptable results as the error in previous bus voltage estimate will go on propagating. However, for systems with "favourable geometry" "good local redundancy" at all the buses and "properly chosen instrument location" reasonably good estimates can be obtained.

4.1 SPLITTING LUMPED EQUATIONS INTO INDEPENDENT EQUATIONS

The line flows at both ends and voltage magnitudes at all the buses are considered to be measured quantities. Like in the 'LO' method, here also the slack bus voltage is assumed to be accurately known. The measurement vector Z is arranged as follows:

(1) The line flows at both the ends of all lines connected to slack bus and the voltage magnitudes of the buses on which these lines terminate come first.

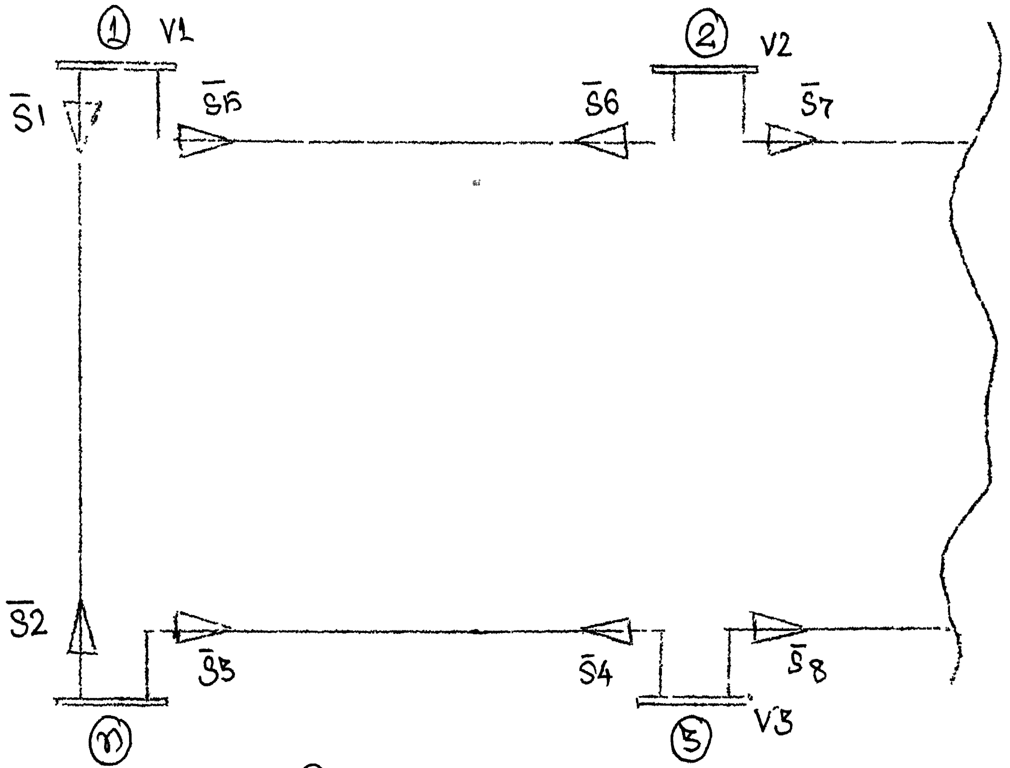

1
V1
2
V2
$\bar{S}1$
$\bar{S}5$
$\bar{S}6$
$\bar{S}7$
$\bar{S}2$
$\bar{S}3$
$\bar{S}4$
$\bar{S}8$
V3
n
3

Fig. 41

(2) Next similar sets of measurements come, for lines connected to the buses on which the previous lines terminate. This is carried out till all the measured lines are considered.

Consider as an example the part of network shown in Fig.4.1. The measurement vector Z consists of line flows $S_1, S_2 \ldots$ etc. and voltage magnitudes $V_1, V_2 \ldots$ etc. This vector is rearranged in the following way:

$$\begin{aligned} Z^T &= (S_1,\ S_2,\ V_1 \ \vdots\ S_3, S_4,\ V_3 \ \vdots\ S_5,\ S_6,\ V_2 \ \vdots\ \ldots) \\ & (\quad Z_1 \quad \vdots \quad Z_2 \quad \vdots \quad Z_3 \quad \vdots\ \ldots) \end{aligned} \tag{4.1}$$

The function $Z = h(X)$ takes the following form.

$$\begin{aligned} Z_1 &= h(X_1,\ V_n) \\ Z_2 &= h(X_3,\ V_n) \\ Z_3 &= h(X_2,\ X_1) \\ &\vdots \end{aligned} \tag{4.2}$$

where V_n is the known complex slack bus voltage and $X_1, X_2 \ldots$ are the complex bus voltages of buses 1,2 ... .

Each of these measurements are independently used to estimate one bus voltage which is treated as constant in subsequent bus voltage estimation. In mathematical terms, this amounts to mean that we obtain $X_1, X_2, \ldots$ sequentially, such that they minimize the local cost functions J_1, J_2, J_3 etc. defined as

$$J_1 = (Z_1 - h(X_1, V_n))^T W(Z_1 - h(X_1, V_n))$$

$$J_2 = (Z_2 - h(X_3, V_n))^T W(Z_2 - h(X_3, V_n)) \tag{4.3}$$

$$J_3 = (Z_3 - h(X_2, X_1))^T W(Z_3 - h(X_2, X_1))$$

$$\vdots$$

Strictly speaking $J_{1\,min}$ and $J_{2\,min}$ and $J_{3\,min}$ and ... etc. does not mean $J(X)_{min}$ (the total cost function defined in WLS method). However, since the local redundancy for each measurement set is high (2.5) results will not deviate much from the WLS solution.

4.2 SOLUTION PROCEDURE

The group of measurements Z_1 is considered first. The two line flows in both ends of the line connected to slack bus to which the group Z_1 corresponds, give two solutions for the voltage of bus at the other end of the line. Third solution to the voltage magnitude at that bus is directly obtained by the measurement. Weighted average is then taken of the three solutions to voltage magnitude and two solutions to phase angle. Assuming this solution to be final one, the solution for the bus voltage of the bus connected to this bus is next carried out in a similar way. Procedure is repeated till all the bus voltages have been obtained.

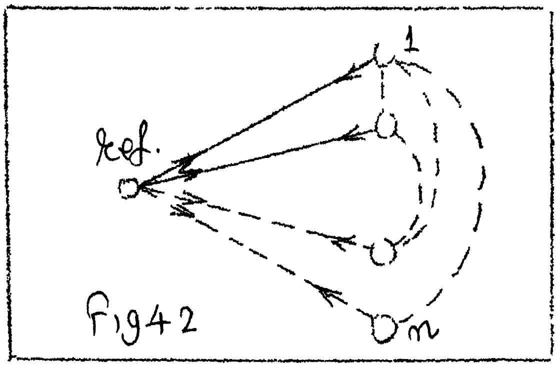


Fig 4 2

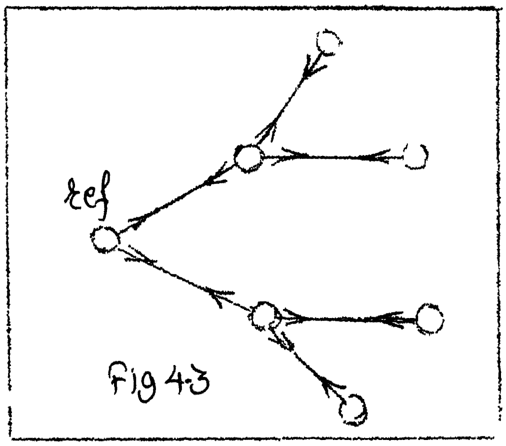


Fig 4.3

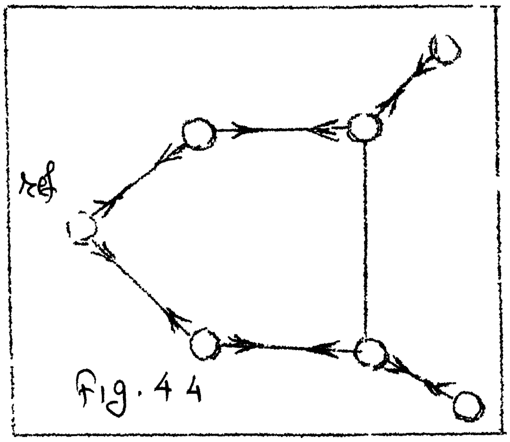


Fig. 4 4

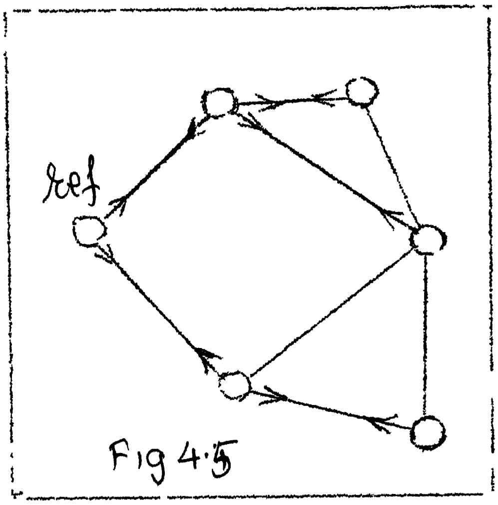


Fig 4.5

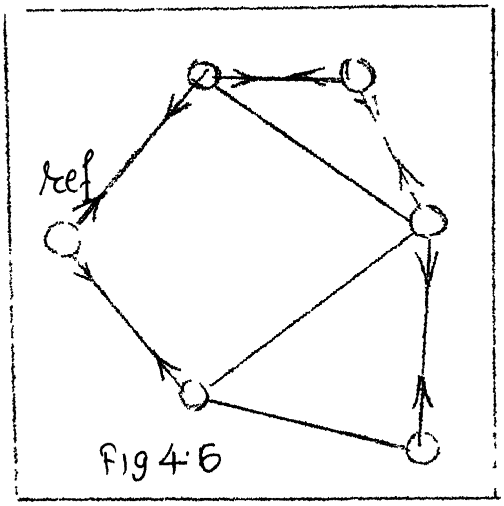


Fig 4.6

Arrows indicate meter locations.

It is clear that the deviation of such a solution from the WLS solution becomes more and more enhanced as the number of measured lines in series go on increasing. Thus a favourable geometry will be one where we have as many lines directly connected to slack bus as possible. An extreme case which of course, is not practicable, is one having all the buses directly connected to the slack bus (Fig.4.2). For such a system the solution obtained by the above method and the WLS method (LO) will be exactly same. Results in both the cases are sensitive to the error in slack bus voltage.

In Figs.4.3 and 4.4, Fig.4.3 will give results more close to the WLS results than will Fig.4.4 give. The importance of meter location in obtaining better results is emphasised. The instrument location of Fig.4.5 is a better choice than that of Fig.4.6 for the same system.

4.3 SOLUTION FOR VOLTAGE FROM LINE FLOWS

The measured line flows from bus 'i' to 'k' and 'k' to 'i' are related with the voltages at buses 'i' and 'k' by

$$P_{ik}+jQ_{ik} = \bar{V}_i \bar{V}_i^* \bar{Y}_{ii}^* + \bar{V}_i \bar{V}_k^* \bar{Y}_{ik}^* \qquad (2.9)$$

$$P_{ki}+jQ_{ki} = \bar{V}_k \bar{V}_i^* \bar{Y}_{ki}^* + \bar{V}_k \bar{V}_k^* \bar{Y}_{kk}^* \qquad (2.10)$$

where $\bar{Y}_{ii}$, $\bar{Y}_{ik}$ ... P_{ik}, Q_{ik} ... $\bar{V}_i \bar{V}_k$ etc. have their usual meanings.

$\bar{V}_k$ is unknown in both these equations. Eqn.(2.9) is linear when voltages are represented in rectangular form. Eqn.(2.10) is nonlinear and has to be solved by N-R method. In rectangular form the solution to $\bar{V}_k$ will be

$$\begin{bmatrix} \Delta P_{ki} \\ \Delta Q_{ki} \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta e_k \\ \Delta f_k \end{bmatrix} \tag{4.4}$$

$$\begin{aligned} e_{k(new)} &= e_k + \Delta e_k \\ f_{k(new)} &= f_k + \Delta f_k \end{aligned} \tag{4.5}$$

where $H = 2e_k\, G_{kk} + e_i G_{ki} - B_{ki} f_i$ (4.6)

$$N = 2f_k\, G_{kk} + e_i\, B_{ki} + G_{ki} f_i \tag{4.7}$$

$$M = 2e_k B_{kk} - e_i B_{ki} + G_{ki} f_i \tag{4.8}$$

$$L = -2f_k B_{kk} - B_{ki} f_i + e_i G_{ki} \tag{4.9}$$

The following terms have been used in expressions for H,N,M and L,

$$\begin{aligned} \bar{Y}_{ii} &= G_{ii} + jB_{ii} \\ \bar{Y}_{ik} &= G_{ik} + jB_{ik} \\ \bar{Y}_{ki} &= G_{ki} + jB_{ki} \\ \bar{Y}_{kk} &= G_{kk} + jB_{kk} \end{aligned} \tag{4.10}$$

$$\begin{aligned} \bar{V}_i &= e_i + jf_i \\ \bar{V}_k &= e_k + jf_k \end{aligned} \tag{4.11}$$

As eqn.(2.9) is linear, solution to $\bar{V}_k$ is obtained by solving this equation and this solution is used as initial guess for the iterative scheme (4.5) for eqn.(2.10).

Solution can also be obtained for $\bar{V}_k$ in polar form. The Jacobian for N-R solution in polar form is given in Section 2.4b. The advantage of solving $\bar{V}_k$ in rectangular form is that we get one linear equation which is easy to be solved and since there are no trigonometric functions involved any where in the Jacobian, convergence is faster than the polar form of solution. Hardware implementation of the rectangular version will be economic compared to polar version as only basic functions of addition, subtraction, multiplication and division are involved in it, whereas, additional software implementation of summation of series or hardware implementation of PROMs or EPROMs is required for SIN and COS functions in the polar form of solution.

4.4 COMPUTER RESULTS

The IEEE 14 bus test system was used to test this method. The results are listed in Table 4.1.

Table 4.1

RESULTS OF INDEPENDENT EQUATION SOLUTION

Bus No.	Load flow solution		Independent equation solution	
	Voltage	Angle	Voltage	Angle
1*	1.06	0.0	1.06	0.0
2	1.045	-4.98	1.045	-4.977
3	1.01	-12.72	1.00994	-12.7177
4	1.019	-10.33	1.02037	-8.7784
5	1.02	-8.78	1.00528	-7.9925
6	1.07	-14.22	1.04252	-20.2528
7	1.062	-13.37	1.0503	-8.8498
8	1.09	-13.36	0.98937	-15.2685
9	1.056	-14.94	1.02575	-13.68259
10	1.051	-15.10	0.9991	-19.22794
11	1.057	-14.79	1.03722	- 9.442
12	1.055	-15.07	1.07872	- 8.85028
13	1.05	-15.16	1.03045	- 9.81833
14	1.036	-16.04	1.01512	-8 .94556

Total time taken : 2.466 sec.

Iterations taken : Max. 3 iteration per line.

CHAPTER 5

DETECTION AND IDENTIFICATION OF GROSS MEASUREMENT ERRORS

State estimation algorithms produce reliable estimate when there is no gross measurement error in the input data. Detection and identification of inadvertant bad data coming in the input data is of great practical importance to enhance operator confidence in the state estimates. A sound acceptance criterion for results required is obtained by statistical analysis of estimates.

5.1 STATEMENT OF THE PROBLEM

In power system model,

$$Z = h(X) + v_z$$

v_z is assumed to be zero mean Gaussian random variable. However, a sudden gross error appearing in the input data violates the assumptions of Gaussian distribution of v_z. The model in such a case is written as

$$Z = h(X) + v$$

where

$$v = v_z + \alpha e \; ; \quad e^T = (0,\ 0,\ \dots\ 1\ \dots\ 0).$$

Entry 1 in e^T corresponds to bad data location or locations and α is the size of bad data. The problem is to detect the presence of bad data and identify which measurement is in gross error.

Detection:

Inspection of the cost function $J(\hat{X})$ of the least squares estimation, at optimal solution $\hat{X}$ reveals whether the input data is in error or not, as bad data is expected to cause an unexpectedly large $J(\hat{X})$. The question of selecting the threshold value of $J(\hat{X})$ is answered by statistical properties of $J(\hat{X})$.

Identification:

Having detected the presence of bad data, identifying which measurement is in error is the next question to be answered. Unfortunately it is not always possible to infer correctly about such bad data location from the residue $(Z-h(\hat{X}))$. The largest residue $(Z-h(\hat{X}))$ need not necessarily coincide with the bad data location. This phenomenon is called smearing. The residual vector $(Z-h(\hat{X}))$ is normalized by variances of its respective components so as to "see through" the smeared residues. The variances of individual entries in the residual vector are obtained by stastical analysis.

5.2 MATHEMATICAL BACKGROUND

The WLS estimate $\hat{X}$ is based on the criterion of minimizing the cost function:

$$J(X) = (Z-h(X))^T R^{-1}(Z-h(X))$$

Thus $$\left.\frac{dJ}{dX}\right|_{X=\hat{X}} = H^T R^{-1}[Z-h(\hat{X})] = H^T R^{-1}\hat{r} = 0 \qquad (5.1)$$

where H is the Jacobian and $\hat{r}$ is the residual vector. $\hat{r}$ can be written as

$$\hat{r} = h(X) + v - h(\hat{X}) \tag{5.2}$$

where X is the true state vector. Defining state estimation error as

$$\delta X = X - \hat{X} \tag{5.3}$$

It follows from (5.3) and (5.1)

$$H^T R^{-1} (h(X) + v - h(\hat{X})) = 0$$

Substituting $h(X) = h(\hat{X}) + H \delta X$;

$$H^T R^{-1} (H \delta X + v) = 0$$

or

$$\delta X = -(H^T R^{-1} H)^{-1} H^T R^{-1} v \tag{5.4}$$

or

$$\delta X = - \Sigma_X H^T R^{-1} v \quad \text{with } \Sigma_X = (H^T R^{-1} H)^{-1} \tag{5.5}$$

with $v = v_z$, i.e. zero mean Gaussian random vector; the covariance matrix of estimation error δX is

$$E[\delta_X \delta_X^T] = \Sigma_X \tag{5.6}$$

The residual vector $\hat{r}$ can be written as

$$\hat{r} = h(X) + v - h(\hat{X}) = W v \tag{5.7}$$

where

$$W = (I - H \Sigma H^T R^{-1}) \tag{5.8}$$

is called residual sensitivity matrix.

The residual covariance matrix becomes

$$E[\hat{r}\ \hat{r}^T] = \Sigma_r = (R - H\ \Sigma_x\ H^T) = W\ R \tag{5.9}$$

The difference between true and estimated measurements is given by

$$\delta h = h(X) - h(\hat{X}) = (W-I)v \tag{5.10}$$

We now define weighted and normalized residuals as

$$\hat{r}_W = \sqrt{R^{-1}}\ \hat{r} \quad \text{and} \quad \hat{r}_N = \sqrt{D^{-1}}\ \hat{r} \tag{5.11}$$

where D is a diagonal matrix obtained by

$$D = \operatorname{diag} \Sigma_r \tag{5.12}$$

The inequality $\hat{r}_N \geq \hat{r}_W$ holds. For $v = v_z$ the covariances of $\hat{r}_W$ and $\hat{r}_N$ are

$$E[\hat{r}_W\ \hat{r}_W^T] = U_W = \sqrt{R^{-1}}\ \Sigma_r\ \sqrt{R^{-1}} \tag{5.13}$$

and

$$E[\hat{r}_N\ \hat{r}_N^T] = U_N = \sqrt{D^{-1}}\ \Sigma_r\ \sqrt{D^{-1}} \tag{5.14}$$

The diagonal elements of U_W and U_N are equal to one.

Define the normalized residual sensitivity matrix W_N as

$$W_N = \sqrt{D^{-1}}\ W \tag{5.15}$$

If $W_{N,jk}$ is the element corresponding to jth row and kth column of W_N then

$$W_{N,jk} = \frac{\Sigma_{r,jk}}{\sqrt{\Sigma_{r,kk}\ \sigma_j^2}} \tag{5.16}$$

where σ_j^2 is the standard deviation of the jth measurement error. Since $\sum_r$ is positive semidefinite, it follows from (5.16) that

$$|W_{N,jj}| \geq |W_{N,jk}| \qquad k=1,2,\ldots,m$$

where m is the size of the measurement vector.

This means that the largest normalized residue r_N coincides with the bad data location. This is not always true with r_W.

Considering the function $J(\hat{X})$, it is easy to see that it has chi-square distribution with k degrees of freedom when $v = v_z$, k = size of the measurement vector(m) - size of the state vector(n).

In the presence of bad data

$$v = v_z + e\,\alpha$$

the residuals $\hat{r}$ and normalized residuals take the form

$$\hat{r} = W\, v_z + W\, e\alpha$$

and (5.17)

$$\hat{r}_N = W_N\, v_z + W_N\, e\alpha$$

Thus $J(\hat{X})$ becomes

$$J(\hat{X}) = v_z^T R^{-1} W v_z + 2\alpha\, e^T R^{-1} v_z + \alpha^2 e^T R^{-1} W e \qquad (5.18)$$

The first term is chi-square distributed, second is normally distributed and third is a constant.

Detection and Identification Theory:

Detection and identification is carried out by hypothesis testing. Two hypothesis HO and H1 are defined.

HO : No bad data

H1 : HO is not true.

(1) The "$J(X)$ Test":

Accept HO if $J(\hat{X}) < \gamma$ (γ = chi-square distribution with k degrees of freedom and specified false alarm probability b defined as the probability of making a wrong decision of rejecting HO when HO is true).

Reject HO, otherwise.

(2) The "r_N test":

Accept HO if $|\hat{r}_{N,k}| < \gamma$

Reject HO, otherwise.

(3) The "r_W test":

Accept HO if $|\hat{r}_{W,k}| < \gamma$

Reject HO, otherwise.

The same threshold γ being used, r_N test is more sensitive to errors since $\hat{r}_N \geq \hat{r}_W$ always.

The following recommendations are made for implementation of bad data detection and identification tests.

(1) Use J(X) test to detect the presence of bad data.

(2) If bad data is detected, apply r_W or r_N test to identify the bad data. r_N test is more accurate but needs off-line calculation of matrix D.

(3) Apply J(X) test to detect the validity of previous state estimate at a given instant of time. Use the new measurements and the old estimate $\hat{X}$ to find J(X). If J(X) is found to be significantly different from the previous J(X) a new state estimation is required. A plot of such a function J(X) with time may be useful for the system behaviour detection. A gradual increase in J(X) will mean that the system state is steadily changing whereas a jump in J(X) will indicate a line outage or a sudden load or generation change.

5.3 IMPLEMENTATION

State estimate based on real power flows in lines and voltage magnitudes was obtained for IEEE 14 bus test system with gross error introduced in one of the real power flow data. The detection and identification of the presence and location of the bad data were done by J(X) and r_N, r_W tests respectively. The variances for all the measurements was taken as 0.01 p.u. and seven redundant measurements were made.

Detection:

J(X) test was applied to detect the presence of bad data. The threshold value of J(X) was obtained from chi-square

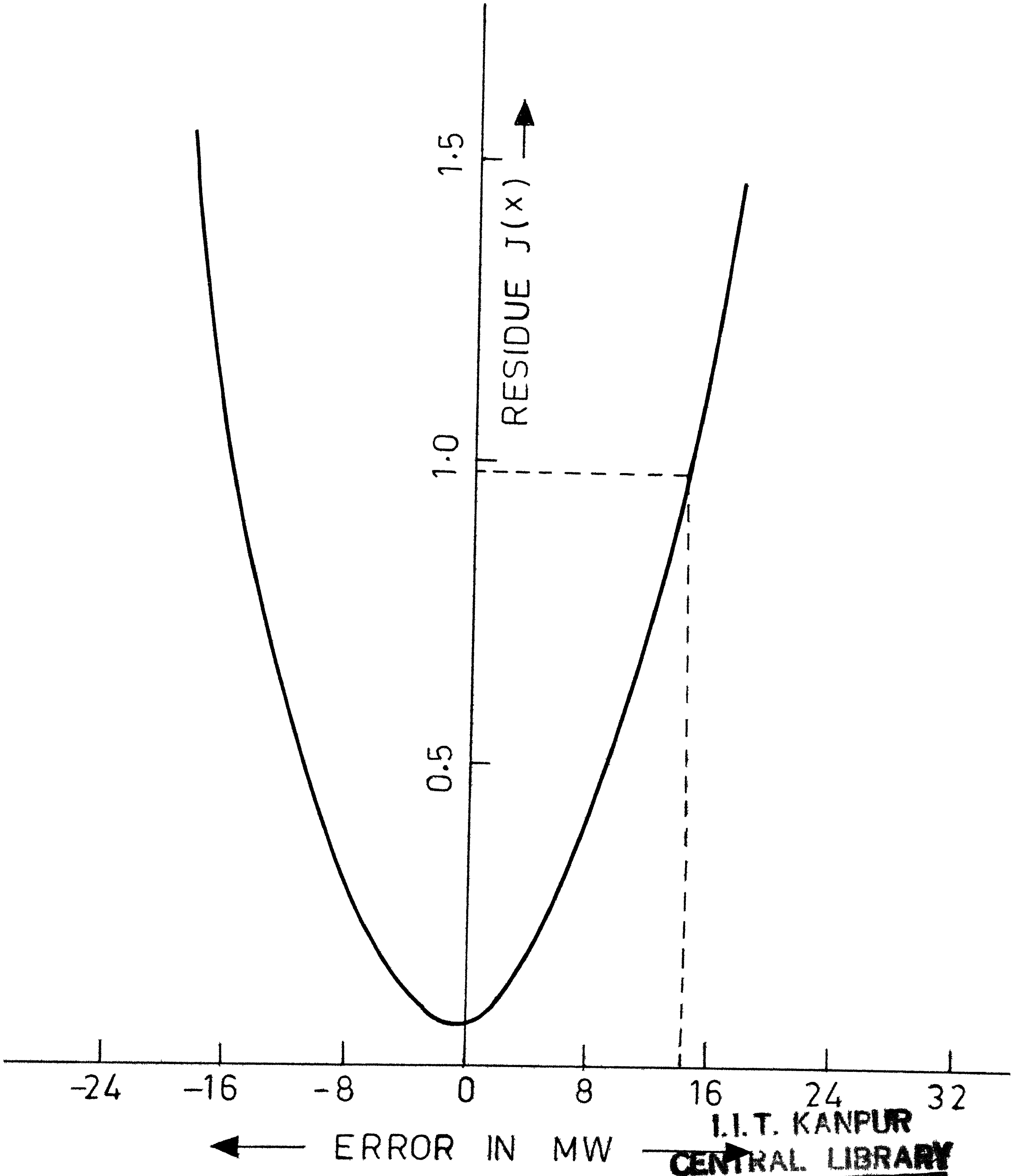


FIG.5.1

distribution tables corresponding to 7 degrees of freedom and false alarm probability b = 0.005. The test could successfully detect the presence of gross measurement error beyond 15 MW. The Fig. 5.1 shows the variation of J(X) with magnitude of error.

Identification:

Identification of the bad data location was done for various cases both by r_N and r_W tests. The r_N test always gave correct results but r_W test in few cases failed to identify correctly. A bad data having been detected and identified was removed from the input data, and the new $J(\hat{X})$ was found to be well within limits of the chi-square value with reduced degrees of freedom and false alarm probability b = 0.005.

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CHAPTER 6

DYNAMIC STATE ESTIMATION

It is often the case in closed loop control systems that the state variables which decide the optimal feedback control law are not directly measurable. Such missing state variables are to be reconstructed from the measurable output variables by observers. Pioneering work in the theory of linear time invariant deterministic and continuous time system observers was done by Luenberger in 1964 [6] . The theory has further been extended to nonlinear time invariant, stochastic and discrete time systems later [36 ,37,41]

6.1 APPLICATION OF OBSERVER THEORY TO POWER SYSTEMS

Concept of such an observer theory to apply optimal control to synchronous machines has been drawing attention of many research workers. Significant improvement in the transient performance of power system can be achieved by applying optimal control law to synchronous machine exciters and governors. An observer can be built to asses the state of a synchronous machine and continuous or discrete feedback control signals can be generated from the knowledge of state vector.

Optimal control of a linear dynamical system:

A linear dynamical system is mathematically written as

$$\dot{x} = A x + B u$$
$$y = C x \qquad (6.1)$$

where x is n state vector, u is r control vector and y is m measurement vector; A is nxn system matrix, B is nxr distribution matrix, C is mxn matrix which relates measurement vector y to the state vector x.

The optimal control law in general is obtained for a system of above kind by choosing u such that the performance index

$$J = \frac{1}{2} \int_0^{\infty} (y^T Q y + u^T R u)dt$$

is minimized, where Q and R are the weighting matrices. Solution u is obtained by minimum principle

$$u^*(t) = -R^{-1} B^T K x(t)$$

where K is symmetric positive definite solution of the matrix Riccati equation

$$KA + A^T(K-K B R^{-1} B^T K + C Q C) = 0$$

The state vector x(t) can be obtained from an observer.

In [10] Arumugam has shown improvement in the transient behaviour of a salient pole machine connected to infinite bus by applying such an optimal control law to excitersand governors using a linear observer. The dynamical model of the synchronous machine was linearized to the standard form of (6.1).

Such a linearized model approach may not always be permissible because the observer of the linearized model is very sensitive to the initial starting value of the unknown state vector x. Also as there is a growing trend towards the use of discrete supplementary excitation control during transient period and use of continuous or discrete control in dynamic state of the synchronous machine, the transient state needs to be accurately assessed which a linearized model may fail to do successfully.

An extended linear observer has been proposed in [23] to assess the state of a synchronous machine more accurately. The observer has been shown to be less sensitive to the starting value of the state vector compared to a linear observer given by Arumugam in [10]. The observation vector y is obtained by a half cycle mean value detecting circuit given in [24]. This circuit holds the average value over half power frequency cycle of the input quantity at its output terminals.

6.2 EXTENDED LINEAR OBSERVER

Nonlinear power system equations are written as

$$\dot{x}(t) = f(x(t)) \tag{6.2}$$

where x is n state vector and f(.) is a nonlinear vector valued function. The m size observation vector y is obtained sequentially at every power frequency cycle by the mean value

detecting circuit of [24]. The nonlinear relation between y and x is

$$y_K = h(x_K) \tag{6.3}$$

where K = 1,2,.... corresponds to period of the power cycle T. The estimated value of state vector $\hat{x}_K$ at interval K according to linear observer theory is given by

$$\hat{x}_{K/K} = \hat{x}_{K/K-1} + K_K(y_K - \hat{y}_{K/K-1}) \tag{6.4}$$

where $\hat{x}_{K/K-1}$ is the predicted value of x_K from the previous estimate at (K-1)th interval, $\hat{y}_{K/K-1}$ is the predicted observation value of y_K from $\hat{x}_{K/K-1}$ and K_K is a suitably chosen filter matrix of nxm size, y_K is the measurement vector at kth interval,

$\hat{x}_{K/K-1}$ is obtained by Taylor series expansion upto second order term as

$$\begin{aligned} x_K &= x(t_{K-1} + T) \\ &\approx x_{K-1} + T\,\dot{x}_{K-1} + \frac{T^2}{2}\,\ddot{x}_{K-1} \\ &\approx x_{K-1} + T\,f\Big|_{x=x_{K-1}} + \frac{T^2}{2}\,\dot{f}\Big|_{x=\bar{x}_{K-1}}\,f\Big|_{x=x_{K-1}} \end{aligned} \tag{6.5}$$

The value of x_K obtained from (6.5) at x_{K-1} is written as $\hat{x}_{K/K-1}$. $\hat{y}_{K/K-1}$ is the value of y_K obtained by substituting $\hat{x}_{K/K-1}$ in eqn. (6.3). The filter matrix K_K which has the properties desired for convergence of eqn. (6.5) is put as

$$K_K = H_K^T\,(H_K H_K^T + C_K)^{-1} \tag{6.6}$$

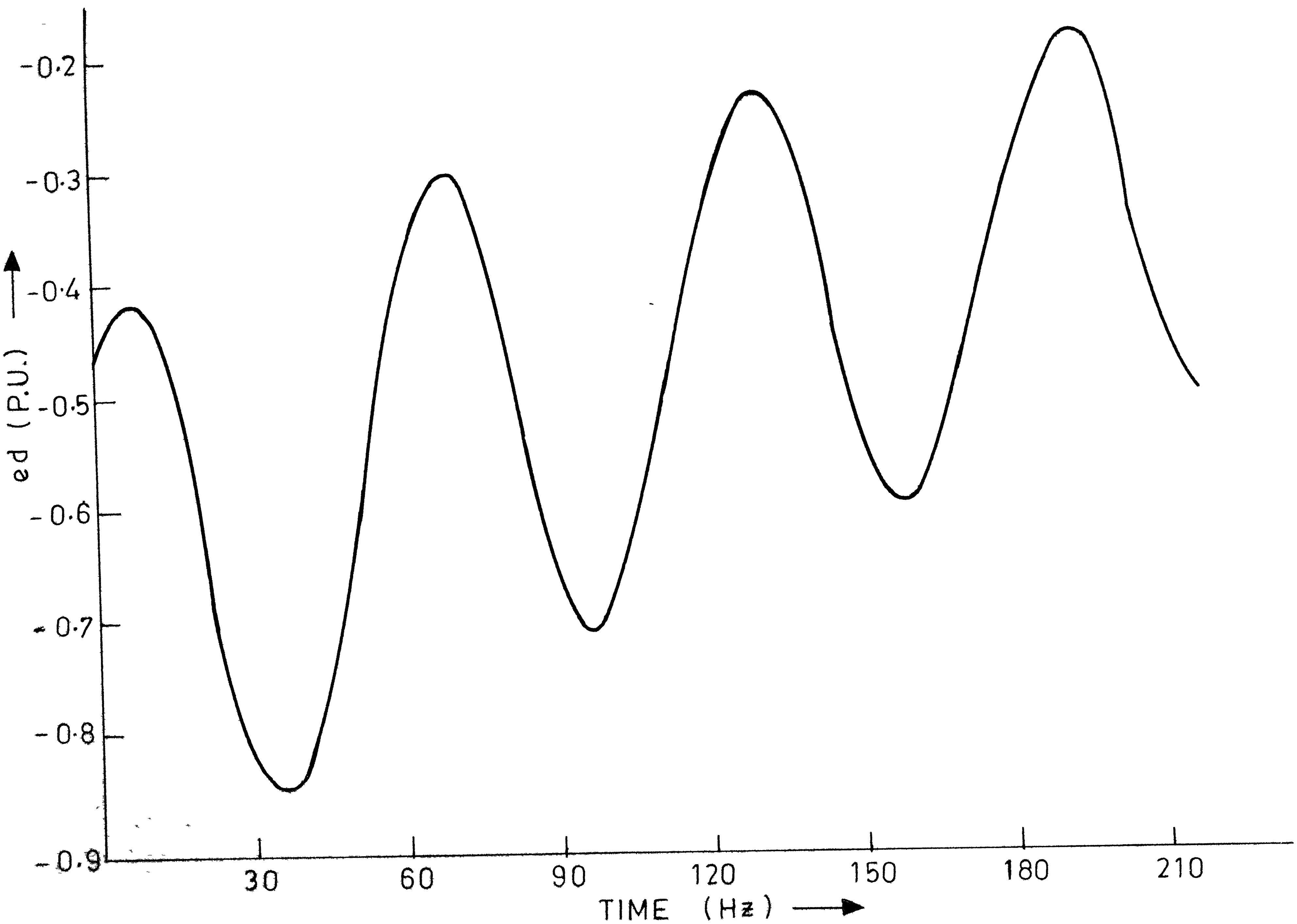

FIG. 6.1(a)

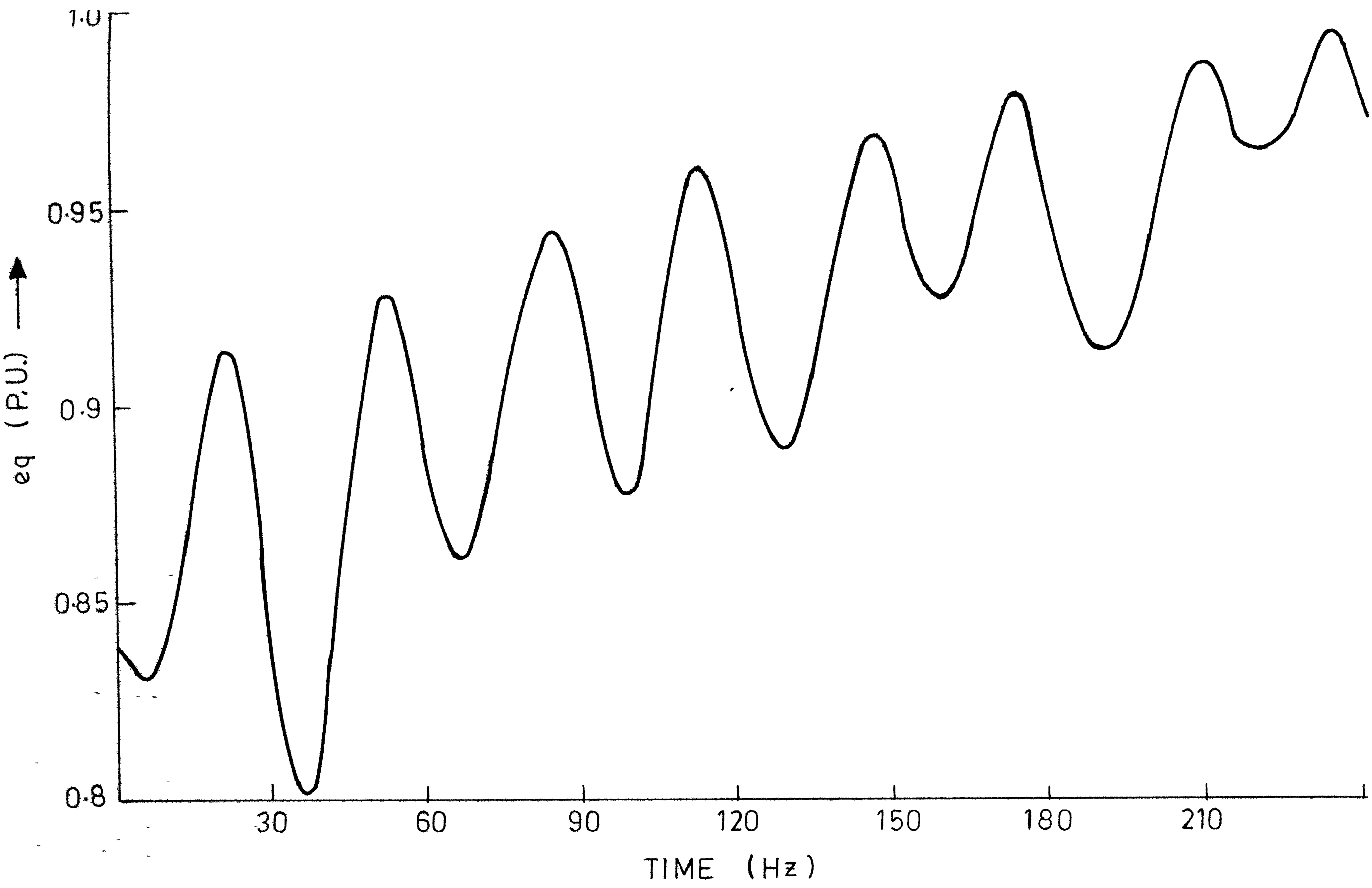

FIG. 6.1 (b)

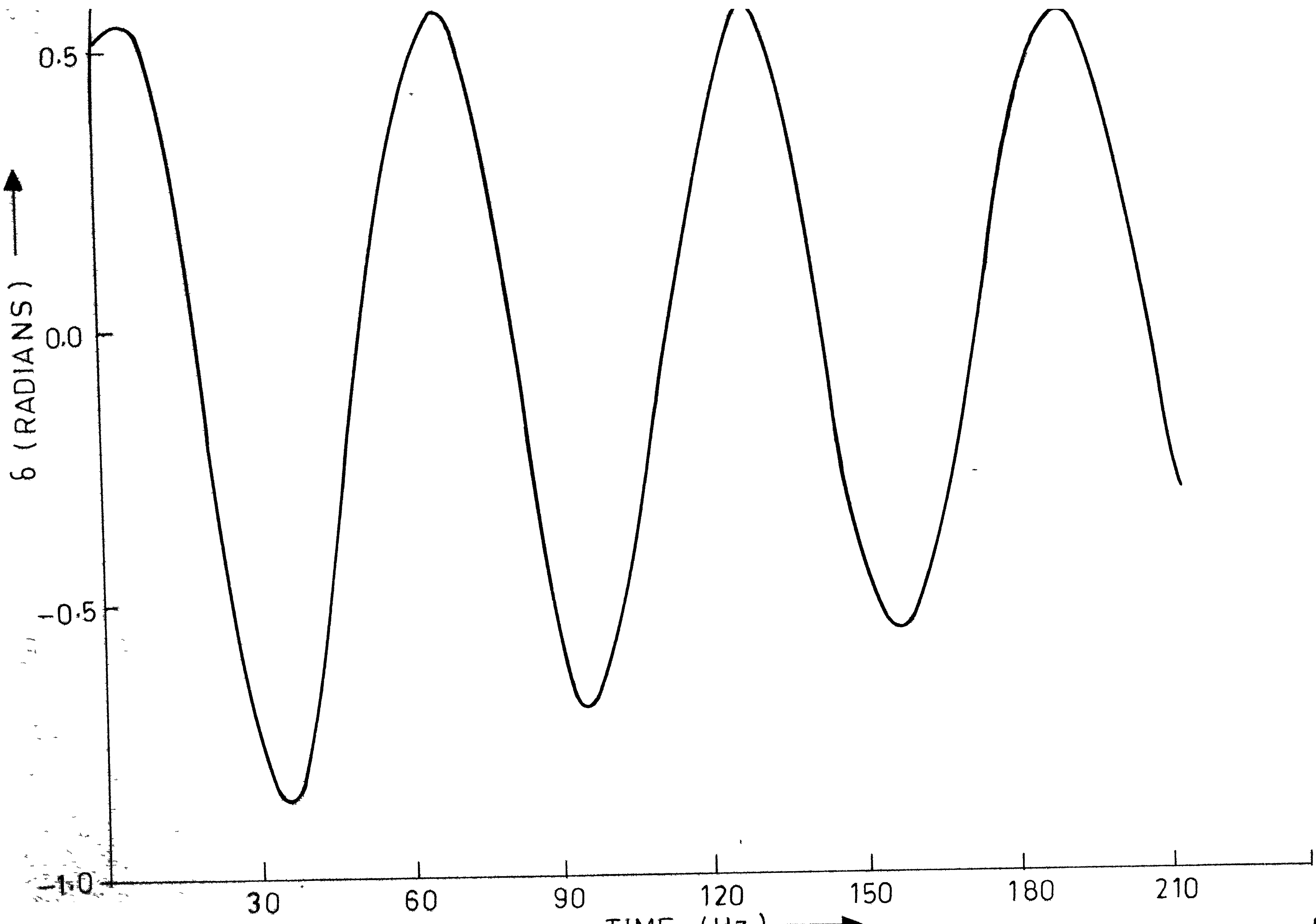

FIG.6.1(c)

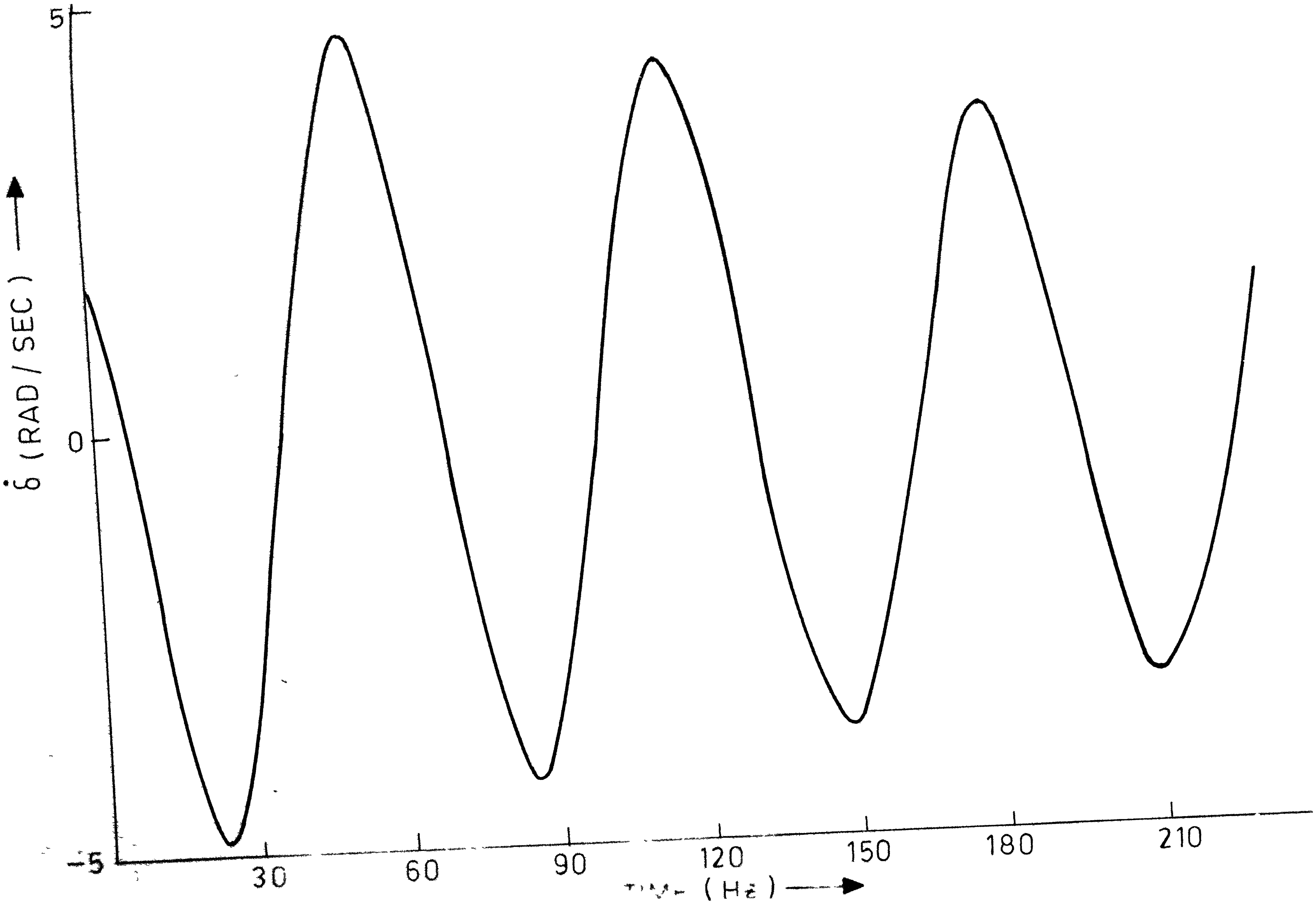

FIG 5.1(d)

where $$H_K = \frac{\partial h}{\partial x^T}\Big|_{x=\hat{x}_{K/K-1}} \tag{6.7}$$

and $$\begin{aligned} [C_K]_{i,i} &= 0.01\,[H_K H_K^T] \\ [C_K]_{i,j} &= 0 \qquad (i \neq j) \end{aligned} \tag{6.8}$$

6.3 IMPLEMENTATION

The above extended linear observer was tested for the cylindrical rotor machine connected to infinite bus given in [23]. The actual transient response was obtained by fourth order R-K method. The results of the R-K muthod were given as input measured quantities y_K to the extended linear observer. The response of the observer is shown in Fig. 6.1.

The observer described in this chapter applies to uncontrolled case. It remains to be demonstrated how this observer can be modified for a controlled case.

CHAPTER 7

FURTHER RESEARCH TOPICS IN STATE ESTIMATION

There are two major class of problems associated with state estimation. First results from the difficulties encountered in carrying out the control applications of static and dynamic estimation successfully. Second problem is related to improvement of the static state estimation accuracy.

7.1 TOPICS RELATED TO STATE ESTIMATION APPLICATIONS

Many potential applications of state estimation to improve static and dynamic performance of power system discussed in Chapter 1 may become offset because of the following practical difficulties:

(1) In an interconnected system the information about the external system is lacking in many respects (network topology, line-parameters, voltage conditions etc.).

(2) Machine models for transient state are inaccurate in the sense that system nonlinearities like saturation etc. are not taken into account by the models. Model parameters change with changing load generation pattern.

The first difficulty offsets the functions of steady state security assessment, contingency evaluation and stability investigation for a simulated critical contingency in own system, whereas the second difficulty impairs the performance of

the optimal controllers used for a synchronous machine. Each of these difficulties form a different class of problems.

External system equivalents for steady state security assessment

The external system equivalencing for steady state security assessment is basically a problem of finding admittance parameters of the reduced equivalent external system. The external system is reduced to managable size and augmented to the own system (O.S.) such that the "buffer zone" is a true representation of external system (E.S.) for a particular load generation pattern and status of the external network. Very few research workers have looked into this problem. Debs is notable among them [16,22]. Debs has recently proposed what seems to be a satisfactory solution to this kind of equivalencing problem [31].

Dynamic equivalents

It is desired to predict the effect of a critical contingency in own system on stability of the overall system. The external system is represented by reduced order dynamic system by lumping the coherent group of generators together in the external system. The coherency analysis and dynamic equivalence is a topic of much research today.

Parameter identification or realization, adaptive control:

The second category of difficulties have to be overcome by applying adaptive control techniques. The parameters of model under consideration are identified or realized and the model adapted continuously. Application of such an identification problem can be found in [19].

7.2 TOPICS RELATED TO IMPROVEMENT IN THE EFFICIENCY OF STATIC STATE ESTIMATORS

Efforts have been made to improve the static state estimators efficiency in terms of accuracy of results.

(1) Choice of meter locations in the system:
Choice of meter locations in the system:

The question of selecting the best choice of meter location to improve the quality of results is an important one which has not been answered satisfactorily so far. However, in [20] some attempt has been made to answer the question of deciding the best measurement to add to an existing measurement set. It follows from linear estimation theory of statistics that the best choice of the matrix H relating observation vector Z with state vector X as $Z = H\,X$ wil will be to have columns of H orthogonal to each other under the condition that $H_i^T H_i$ = constant, where H_i is the ith column of H [40]. Selecting H is not in our hands for power system problems but one could select the best measurement to add to a set of existing measurements by checking orthogonality of columns of H for each of the choices of measurements at disposal. It is proved in [20] that most orthogonal vector

to the columns of a matrix H corresponds to eigenvector associated with the smallest eigenvalue of the matrix $(H^T H)$. Use is made of the above theorem in [20] to decide best measurement to add to a set of measurements but the [2,14] computations involved are prohibitively large. A considerable amount of work needs to be done in this area.

Kalman filtering:

Recursive Kalman filters have been applied to state estimation by modelling the power system as

$$x_{n+1} = \phi[x_n] + w_n$$

$$y_{n+1} = \gamma(x_n)$$

where w_n is a stochastic process of known statistics. This needs a statistical load modelling of the power system loads at different intervals of time. Kalman filtering solution of state estimation is more realistic in the sense that the power system is represented by dynamical equations. Accurate knowledge of statistics of the stochastic process w_n is a pre-requisite for Kalman filter solution.

CHAPTER 8

EVALUATION OF DIFFERENT ALGORITHMS AND CONCLUSION

The static state estimation algorithms discussed in Chapters 2, 3 and 4 are compared in this chapter in terms of computational time, storage requirements and their merits and limitations. Each of these methods is evaluated below.

8.1 INDEPENDENT EQUATION SOLUTION METHOD

It is seen from the Table 8.1 that this method requires minimum time and storage (In fact the storage requirements can further be reduced as there were many variable names used for the sake of clarity of the programme). Basically this method gives an approximate solution to state estimation and accuracy of results is largely dependent upon system configuration. In spite of the lim tation this method has, there is a distinct advantage of this method over others that it can be implemented on a small computer. It does not involve matrix storage and trigonometric functions. Hence memory size and system software and hardware for the computer are minimum.

This method assumes that line flows at both ends are available. However, if one end line flow is missing estimation should be carried out with only one end line flow instead of putting the missing end line flow equal to the other end line flow. Alternatively if the bus injection

measurement at the missing end bus is available the missing line flow can be reconstructed and used for estimation. Bus injection if available could also be used to check which end line flow measurement is to be rejected when the two end line flows are found to be significantly different from each other and one of them is thought to be incorrect.

An improvement in the results could possibly be obtained by taking average of the solutions obtained by tracing alternative paths. It is of course not possible to trace all possible paths and to obtain all possible sets of solutions but a properly selected group of paths will give good enough results.

8.2 'LO' ALGORITHM

This method is seen to have taken next lowest memory storage and computer time. The only approximation involved in derivation of this algorithm is that the weights to the P and Q measurements at one end of line flow are assumed to be equal.

The main advantage of this algorithm is that it is very easy to build the gain matrix and to modify it for changes in system configuration. The gain matrix is constant and sparse in nature. When e ual weights are given all measurements, i.e. W is taken as identity the off-diagonal entries of the gain matrix become 0, 1 or -1. Individual

memory bits can be utilized to store these entries instead of storing them in a single word. Such storing and retrieving of individual bits can be done by AND and OR functions of FORTRAN which are available in most of the modern computers.

When these techniques for efficient storage are applied to 'LO' method, it becomes more powerful. Moreover the time taken to converge given in Table 8.1 for this method is rather misleading, it will take lesser time in on line mode of operation. The gain matrix does not change unless the system configuration or meter location itself changes. Thus the inversion or triangular factorization of the gain matrix which is the most time consuming part of the algorithm is obviated in real time mode of operation.

It is assumed in this method that the reference bus voltage is correctly known. The error in it affects the results.

8.3 'PV' METHOD

This method can be seen from Table 8.1 to have moderate memory and time requirements. This method is not different from a lumped WLS method. The objective to test this method was to demonstrate the feasibility of solution from real line flow and bus voltage measurement data. This method is particularly suited for Indian conditions as the data it takes is readily available and the additional cost of equipment is not required.

8.4 'FDS' AND 'LUMPED WLS' ALGORITHM

Fast decoupling approach basically is used to reduce the memory and convergence time of lumped WLS method. From the Table 8.1, it can been that FDS does take lesser time than a lumped WLS method but no such improvement in total memory storage is apparent. This is because of the fact that the sample system on which these methods have been implemented is very small in size. The data storage in lumped WLS is approximately proportional to n^2 for an n bus system to $n^2/2$ for FDS method.

Unlike 'LO' method, FDS and lumped WLS methods can take any number of measurement and a very high figure of redundancy can be obtained with them.

Table 8.1

Comparison of different methods.

	'LO' method	Lumped WLD method	FDS method	PV method	Independent equation solution
Data storage	1.7 K	5 K	7.7 K	3.6 K	0.296 K
Programme storage	1.8 K	3.9 K	4.2 K	2 K	0.924 K
Total storage	3.5 K	8.9 K	9.9 K	5.6 K	1.22 K
Time taken	7.8 sec.	133 sec.	99.66 sec.	86.65 sec.	2.466 sec.
Total no.of iterations	5	4	12	4	Max.3 iterations per line

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APPENDIX

LEAST SQUARES ESTIMATION:

Consider a static system in which an observable variable vector z of size m is connected with n state vector x.

$$z = H\, x \qquad (A.1)$$

where H is a known mxn matrix. It is desired to solve for x from given z and H. Assume that $m \geq n$. If all the observations were to be correct, eqn.(A.1) would be a set of consistent equations, i.e. any n equations out of the m would have given the same solution to x. However if(A.1) is not a set of consistant equations, the vector x is chosen which minimizes the performance index

$$J = (z - H\, x)^T\, (z - H\, x) \qquad (A.2)$$

or $\quad dJ/dx = (-H)^T\, 2(z - H\, x) = 0$

$$\hat{x} = (H^T\, H)^{-1}\, H^T\, z \qquad (A.3)$$

WEIGHTED LEAST SQUARES ESTIMATION:

Consider a case where the observations z are known to be contaminated by a random noise .

$$z = H\, x + \eta \qquad (A.4)$$

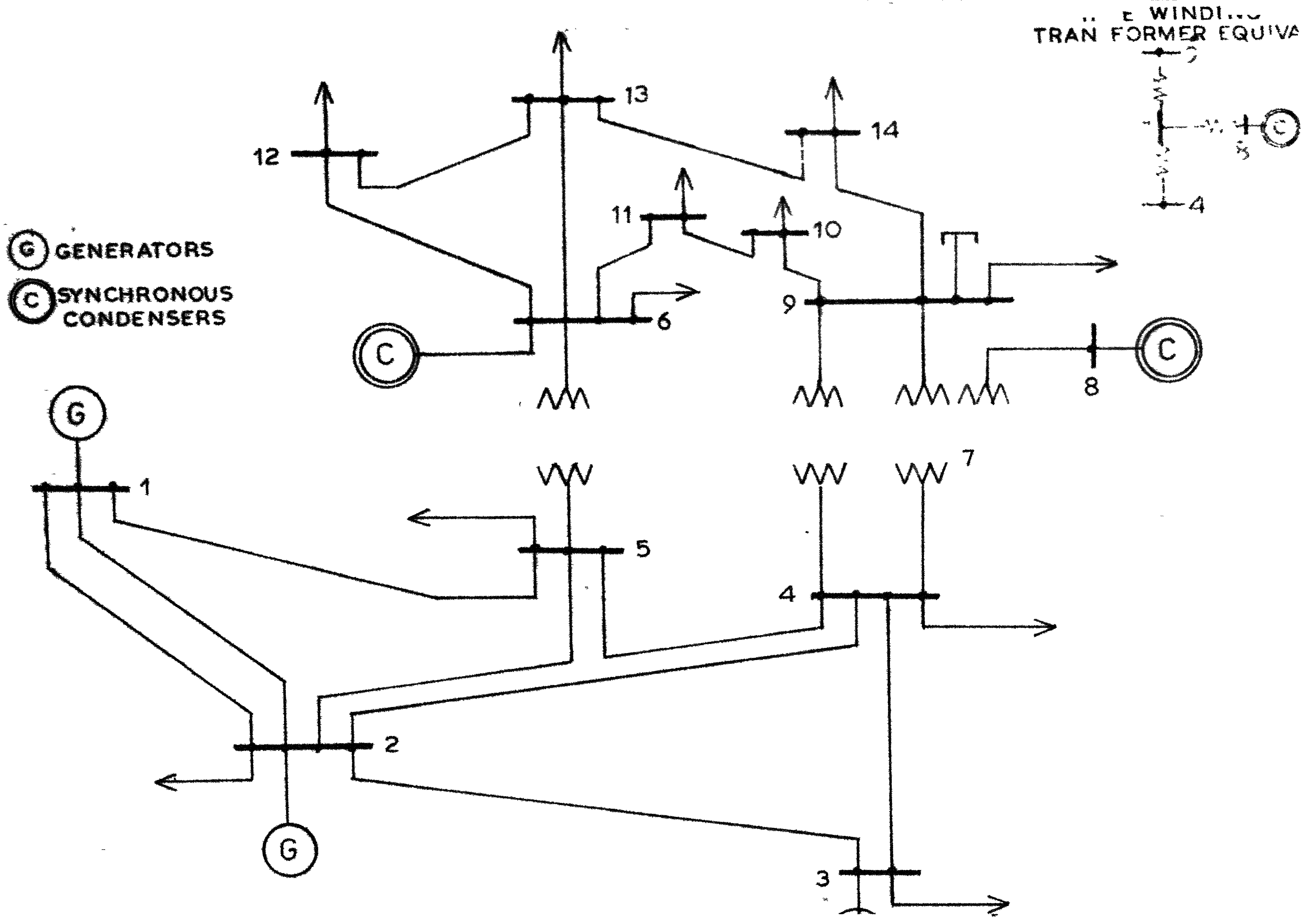
Estimation of state x is done by the minimization of weighted sum of squares of errors by giving weights to each measurements of z, as follows

$$J = (z-Hx)^T\, W(z-Hx) \qquad (A.5)$$

where W is the inverse of error covariance of the noise .
It becomes a diagonal matrix when the noise vector is uncorrelated.

The stimate of x can be obtained similar to(A.3) as follows

$$\hat{x} = (H^T W H)^{-1} H^T W(z-Hx) \tag{A.6}$$



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APPENDIX B

IEEE 14 BUS TEST SYSTEM

The 14 bus test system shown in Fig.

Table B.1: Impedance and line charging data.

Line Designation	Resistance p.u.	Reactance p.u.	Line charging p.u.
1 - 2	0.01938	0.05917	0.0261
1 - 5	0.05403	0.22304	0.0246
2 - 3	0.04699	0.19797	0.0219
2 - 4	0.05811	0.17632	0.0187
2 - 5	0.05695	0.17388	0.0170
3 - 4	0.06701	0.17102	0.0173
4 - 5	0.01335	0.04211	0.0064
4 - 7	0.0	0.20912	0.0
4 - 9	0.0	0.55618	0.0
5 - 6	0.0	0.25202	0.0
6 - 11	0.09498	0.19890	0.0
6 - 12	0.12291	0.25581	0.0
6 - 13	0.06615	0.13027	0.0
7 - 8	0.0	0.17615	0.0
7 - 9	0.0	0.11001	0.0
9 - 10	0.03181	0.08450	0.0
9 - 14	0.12711	0.27038	0.0
10 - 11	0.08205	0.19207	0.0
12 - 13	0.22092	0.19988	0.0
13 - 14	0.17093	0.34802	0.0

Impedance and line charging susceptance is per unit on a 100,000 kva base.

Line charging one-halfof total charging of line.

Table B.2: Generation and Load Schedules.

Bus Number	Generation MW	Generation MVAR	Load MW	Load MVAR
1*	0.0	0.0	0.0	0.0
2	40.0	0.0	21.7	12.7
3	0.0	0.0	94.2	19.0
4	0.0	0.0	47.8	-3.9
5	0.0	0.0	7.6	1.6
6	0.0	0.0	11.2	7.5
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
9	0.0	0.0	29.5	16.6
10	0.0	0.0	9.0	5.8
11	0.0	0.0	3.5	1.8
12	0.0	0.0	6.1	1.6
13	0.0	0.0	13.5	5.8
14	0.0	0.0	14.9	5.0

* indicates slack bus.

Table B.3: Transformer Data.

Transformer designation	Tap Setting
4 - 7	0.978
4 - 9	0.969
5 - 6	0.932

Table B.4: Static Capacitor Data.

Bus Number	Susceptance p.u.
9	0.19

Table B.5: Load Flow Solution

Bus		Line Flow	
From	To	MW	MVAR
1	2	156.8	-20.4
1	5	75.6	3.5
2	1	-152.5	27.6
2	3	73.2	3.6
2	4	56.2	-2.3
2	5	41.5	0.7
3	2	-70.9	1.6
3	4	-23.3	2.8
4	2	-54.5	3.4
4	3	23.7	-5.4
4	5	-61.2	15.6
4	7	28.0	-9.4
4	9	16.1	-0.3
5	1	-72.8	2.6
5	2	-40.6	-1.6
5	4	61.7	-15.3
5	6	44.0	12.8
6	5	-44.0	-8.4
6	11	7.3	3.5
6	12	7.8	2.5
6	13	17.7	7.2
7	4	-28.0	11.1
7	8	0.0	-16.9
7	9	28.1	6.8
8	7	0.0	17.4
9	4	-16.1	1.6
9	7	-28.1	-5.0
9	10	5.2	4.4
9	14	9.5	3.7
10	9	-5.2	-4.3
10	11	-3.7	-1.6
11	6	-7.4	-3.4
11	10	3.7	1.6
12	6	-7.7	-2.4
12	13	1.6	0.7
13	6	-17.5	-6.8
13	12	-1.6	-0.7
13	14	5.6	1.7
14	9	-9.3	-3.4
14	13	-5.6	-1.6

Table B.5 (Continued)

Bus	Voltage		Generation		Load	
	Magnitude	Angle	MW	MVAR	MW	MVAR
1	1.06	0.0	232.4	-16.2	0.0	0.0
2	1.045	-4.98	40.0	42.4	21.70	12.70
3	1.01	-12.72	0.0	33.4	94.2	19.00
4	1.019	-10.33	0.0	0.0	47.8	-3.9
5	1.02	-8.78	0.0	0.0	7.6	1.6
6	1.07	-14.22	0.0	12.2	11.20	7.50
7	1.062	-13.37	0.0	0.0	0.0	0.0
8	1.09	-13.36	0.0	17.4	0.0	0.0
9	1.056	-14.94	0.0	0.0	29.50	16.60
10	1.051	-15.10	0.0	0.0	9.0	6.8
11	1.057	-14.79	0.0	0.0	3.50	1.80
12	1.055	-15.07	0.0	0.0	6.10	1.6
13	1.05	-15.16	0.0	0.0	13.50	5.8
14	1.036	-16.04	0.0	0.0	14.9	5.0

DATA INPUT TO VARIOUS STATE ESTIMATION ALGORITHMS:

1. Lumped WLS and FDS Method

Bus injections at all buses, line flows in one direction of all lines and voltage magnitudes at all buses were used as input data for both lumped WLS and FDS methods. Following is the description of lines considered.

LINE FLOWS							
From	To	From	To	Fro	To	From	To
1	2	3	4	6	11	9	10
1	5	4	5	6	12	9	14
2	3	4	7	6	13	10	11
2	4	4	9	7	8	12	13
2	5	5	7	7	9	13	14

2. PV Method

The real power flow in the lines (one end only) considered in the FDS and lumped WLS method and voltage magnitudes at all buses were given as input data to PV algorithm.

3. LO Method

Line flows in both the directions of the following lines was the input to this algorithm.

LINES							
From	To	From	To	From	To	From	To
1	2	4	7	7	9	13	14
1	5	5	6	9	10	4	5
2	3	6	12	10	11	2	4
3	4	7	8	12	13	9	14

4. Independent Equation Solution:

The line flows in both directions of the following lines was the input data.

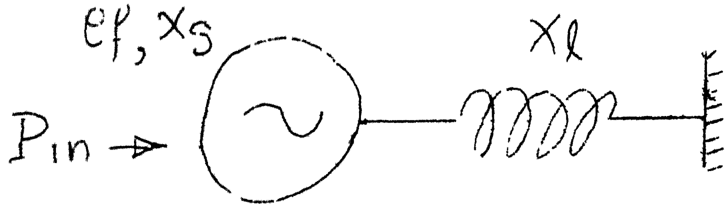
From	1	2	2	2	4	4	9	9	5	6	6	6	7
To	2	3	4	5	7	9	10	14	6	12	13	11	8

APPENDIX C

DESCRIPTION OF THE SYNCHRONOUS MACHINE USED FOR TRANSIENT STATE ESTIMATION PURPOSE

The following assumptions were made for the synchronous machine used for transient state estimation studies ~~xy~~by the extended linear observer.

1. Cylindrical rotor with no damper windings.
2. Two fields with same structure placed on the d-and q-axis.
3. No saturation
4. Transformer voltage, armature resistance and line resistance neglected.



Synchronous machine connected to infinite bus

Nomenclature

T_{do}	-	field open circuit time constant
X_s	-	synchronous reactance
X_l	-	line reactance
T_{dl}	-	field short circuit time constant
e_{fd} & e_{fq}	-	direct and quadrature axis component of the field excitation voltage
w	-	angular speed rad/sec
P_{in}	-	mechanical power input
H	-	inertia time constant (sec.)

M - moment of inertia (= 2H/w)

D - damping coefficient.

Model of the Synchronous Machine

State vector x consists of

$$x = (x_1, x_2, x_3, x_4)^T = (e_d, e_q, \delta, \dot{\delta})^T \qquad (C.1)$$

The model $\dot{x} = f(x)$ is written as

$$\begin{aligned}
\dot{x}_1 &= -A_1x_1 + A_2x_4\cos x_3 + A_3\sin x_3 - A_4 \\
\dot{x}_2 &= -A_1x_2 - A_2x_4\sin x_3 + A_3\cos x_3 + A_5 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= A_7x_4 + A_8(x_2 \sin x_3 - x_1\cos x_3) + A_6
\end{aligned} \qquad (C.2)$$

where

$$\begin{aligned}
&A_o = T_{do} + \frac{x_s}{x_1} T_{d1}; \quad A_1 = (1 + \frac{x_s}{x_1})/A_o ; \\
&A_2 = \frac{x_s}{x_1} T_{d1}\ e_s/A_o ; \quad A_3 = x_s/x_1\ e_s/A_o ; \\
&A_4 = \frac{e_{fq}}{A_o} ; \quad A_5 = e_{fq}/A_o ; \quad A_6 = P_{in}/M ; \\
&A_7 = -D/M ; \quad A_8 = -\frac{e_s}{x_1} /M
\end{aligned} \qquad (C.3)$$

The output vector y consists of terminal voltage y_1, output power y_2 speed deviation y_3. The relation $y = h(x)$ is written as:

$$\begin{aligned}
y_1 &= (x_1^2 + x_2^2)^{\frac{1}{2}} \\
y_2 &= \frac{e_s}{x_1} (x_2 \sin x_3 - x_1 \cos x_3) \\
y_3 &= x_4
\end{aligned} \qquad (C.4)$$

Systrm constants

$x_s = 1$, $x_1 = 0.6$, $T_{do} = 3$ sec., $T_{d1} = 1$ sec.

$H = 5$ sec., $D = 0.005$, $w = 2\pi(60)$ rad/sec., $e_s = 1$,

$e_{fd} = e_{fq} = 1$, $P_{in} = 0.76923$.

Initial values of the state vector

$e_{do} = -0.48593$, $e_{qo} = 0.86149$, $\delta_o = 0.507635$

$$\dot{\delta}_o = 2.1571.$$

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